NEXT IAS

(To be filled by candidate)

Name of Candidate : BHASWATA SALKIA

Roll No. : .....

Registration Number : .....

Exam Centre : Old Rajinder Nagar ☐ Bhopal ☐ Online ☑ Date of Examination : 21/8/2023

CSE (MAINS) TEST SERIES - 2023

Test-01

Dated : 18-06-2023

MATHEMATICS OPTIONAL

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

*There are **EIGHT** questions divided in **Two sections** and printed in **ENGLISH**.*

*Candidate has to attempt **FIVE** questions in all.*

***Question No. 1 and 5** are compulsory and out of the remaining, any **THREE** are to be attempted choosing at least **ONE** question from each section.*

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission. Certificate which must be stated clearly on the cover of this Question-cum- Answer (QCA). Booklet in the space provided.

No marks will be given for answers written in a medium other than the authorized one.

Word limit in questions, wherever specified, should be adhered to.

Attempts of question shall be counted in sequential order. Unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA). Booklet must be clearly struck off.

Scanned with OKEN Scanner

(For filling by Examiners only)

Q. No.	Page No.	Max. Marks	Marks	Total	Signature
1. (a)	4	10			
1. (b)	6	10			
1. (c)	8	10			
1. (d)	10	10			
1. (e)	12	10			
2. (a)	14	20			
2. (b)	18	15			
2. (c)	21	15			
3. (a)	24	15			
3. (b)	27	20			
3. (c)	32	15			
4. (a)	35	10			
4. (b)	37	20			
4. (c)	41	20			
5. (a)	45	10			
5. (b)	47	10			
5. (c)	49	10			
5. (d)	51	10			
5. (e)	53	10			
6. (a)	55	15			
6. (b)	58	20			
6. (c)	62	15			
7. (a)	65	15			
7. (b)	68	15			
7. (c)	71	20			
8. (a)	75	20			
8. (b)	78	15			
8. (c)	81	15			
			Grand Total		

Remarks

Observations:

Scanned with OKEN Scanner

SECTION-A

Q.1 (a) Let $u_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix}, u_2 = \begin{pmatrix} 2 \\ -1 \\ -5 \\ 2 \end{pmatrix}, u_3 = \begin{pmatrix} 1 \\ -1 \\ -4 \\ 0 \end{pmatrix}$. Find the value of h so that $b = \begin{pmatrix} 2 \\ 1 \\ 1 \\ h \end{pmatrix}$ lie in span $\{u_1, u_2, u_3\}$. (10)

Ans: If b lies in the span of u_1, u_2, u_3

If will also lie in the span of u_1, u_2, u_3

in row reduced form

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & -5 & 2 \\ 1 & -1 & -4 & 0 \end{bmatrix} \begin{matrix} \\ R_2 = R_2 - 2R_1 \\ R_3 = R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -9 & -6 \\ 0 & -2 & -6 & -4 \end{bmatrix} \begin{matrix} \\ R_2 = -\frac{1}{3}R_2 \\ R_3 = R_3 - \frac{2}{3}R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, let $b = \begin{pmatrix} 2 \\ 1 \\ 1 \\ h \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 3 \\ 2 \end{pmatrix}$

So, $\alpha = 2$,

$\alpha + \beta = 1$

$\alpha + 3\beta = 1 \Rightarrow \beta = 0$

$h = 4\alpha + 2\beta$

As the equations are inconsistent, no value of h, the vector b lies on the span

Candidates must not write on this margin

Aug matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & h \end{array}\right] \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - 2R_1 \\ R_4 = R_4 - 4R_1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & h-8 \end{array}\right]$$

now 2 & 3 are consistent

as $3R_2 = R_3$

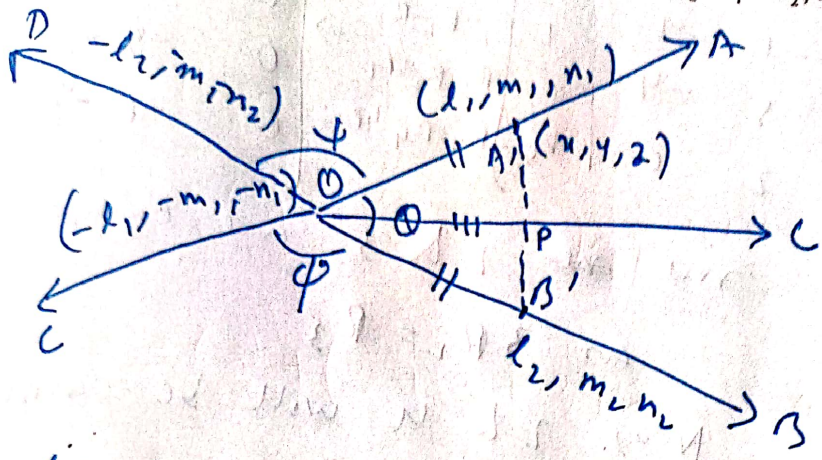
Now, 2 & 4 will be consistent

if $2R_2 = R_4$

$\Rightarrow -2 = h - 8$

$\Rightarrow h = 6$ //

Q.1 (b) (l_1, m_1, n_1) and (l_2, m_2, n_2) are the dc's of two concurrent lines, show that the dc's of the two lines bisecting the angles between them are proportional to $(l_1 \pm l_2, m_1 \pm m_2, n_1 \pm n_2)$. (10)



Let two points be considered at equal distance of t along OA & OB.
Let OC be the angular bisector.
The co-ordinates of those points are

~~(tl_1, tl_2, tl_3) &~~

$A'(tl_1, tm_1, tn_1)$ & $B'(tl_2, tm_2, tn_2)$

Now, triangle OA'P & OB'P are congruent.

So, P is the midpoint of A'B'

Hence co-ordinate of P is

$t(l_1+l_2), t(m_1+m_2), t(n_1+n_2)$

As, t ranges from 0 to infinity. it covers all points of OC

Hence its direction cosines are

Candidates must not write on this margin

proportional to $l_1 l_2, m_1 m_2, n_1 n_2$

similarly for unit angle ψ

by similar procedure

the dc will be proportional to

$l_1 - l_2, m_1 - m_2, n_1 - n_2$

Hence proved

Q.1 (c) If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$, find $A^7-7A^6+15A^5-12A^4-A^3+8A^2-15A-4I$. **(10)**

The characteristic eqn is

$$\lambda^3-6\lambda^2+(\Sigma \text{minor})\lambda-\det A=0$$

$$=\lambda^3-6\lambda^2+(3+3+3)\lambda-4=0$$

$$=\lambda^3-6\lambda^2+9\lambda-4=0$$

By Cayley Hamilton theorem

$$A^3-6A^2+9A-4I=0 \quad -(1)$$

dividing

$$\begin{array}{r|l} & A^4-A^3+A+I \\ \hline A^3-6A^2+9A-4I & A^7-7A^6+15A^5-12A^4-A^3+8A^2-15A-4I \\ & A^7-6A^6+9A^5-4A^4 \\ \hline & -A^6+6A^5-8A^4-A^3 \\ & -A^6+6A^5-9A^4+4A^3 \\ \hline & A^4-5A^3+8A^2-15A \\ & A^4-6A^3+9A^2-4A \\ \hline & A^3-A^2-11A-4I \\ & A^3-6A^2+9A-4I \\ \hline & 5A^2-20A \end{array}$$

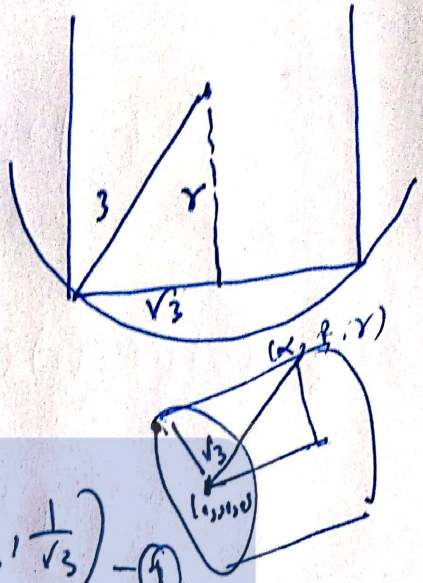
using (1)

$A^7-7A^6+\ldots-4I$ is equivalent to

$$5A^2-20A=\begin{bmatrix}-10 & -5 & 5\\ -5 & -10 & -5\\ 5 & -5 & -10\end{bmatrix} //$$

Scanned with OKEN Scanner

Q.1 (d) Find the equation of the right circular cylinder which passes through the circle $x^2+y^2+z^2=9, x-y+z=3$. **(10)**



distance of the plane from the centre

$$\frac{|-3|}{\sqrt{3}} = \sqrt{3} \quad -(1)$$

Hence $r = \sqrt{3}$

d.c of axis $= \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ — (2)

Now, let α, β, γ lies on the cylinder.

So, $\alpha^2+\beta^2+\gamma^2 = (\sqrt{3})^2 + \left((\alpha-0)\frac{1}{\sqrt{3}} + (\beta-0)\frac{-1}{\sqrt{3}} + (\gamma-0)\left(\frac{1}{\sqrt{3}}\right)\right)^2$

$\Rightarrow \alpha^2+\beta^2+\gamma^2 = 3 + \frac{1}{3}(\alpha - \beta + \gamma)^2$

$\Rightarrow \alpha^2+\beta^2+\gamma^2 = 3 + \frac{1}{3}(\alpha^2+\beta^2+\gamma^2-2\alpha\beta-2\beta\gamma+2\alpha\gamma)$

$\Rightarrow 2\alpha^2+2\beta^2+2\gamma^2-9 = 2\alpha\gamma-2\alpha\beta-2\beta\gamma$

replacing α, β, γ with (x, y, z)

the required cylinder is

$$2x^2+2y^2+2z^2-2xz+2xy+2yz-9=0$$

Q.1 (e) Find the equation of the plane which cuts the paraboloid $x^2 - 3y^2 = 3z$ in the conic with centre $(1, 2, 3)$. **(10)**

The required eqⁿ of plane is

$$S_1 = T$$

$$(1)^2 - 3(2)^2 - 3(3) = x(1) - 3y(2) - \frac{3}{2}(z+3)$$

$$\Rightarrow 1 - 12 - 9 = x - 6y - \frac{3}{2}z - \frac{9}{2}$$

$$\Rightarrow -20 = x - 6y - \frac{3}{2}z - \frac{9}{2}$$

$$\Rightarrow -\frac{31}{2} = x - 6y - \frac{3}{2}z$$

$$\Rightarrow x - 6y$$

$$\Rightarrow 2x - 12y - 3z + 31 = 0$$

is the req plane

Q.2 (a) If A and B are Hermitian matrices such that $A^2 + B^2 = 0$, show that $A = B = 0$. (20)

A, B Hermitian so

$A^\theta = A$

$B^\theta = B$

let A & B are of size $n \times n$

Now, $A = (a_{ij})$

given hermitian

So. $(a_{ij}) = (\overline{a_{ji}})$

Ily $(b_{ij}) = (\overline{b_{ji}})$

Now, $A^2 + B^2 = 0$

So, $(A^2 + B^2)_{ii'} = 0$

$$\Rightarrow \sum_{k=0}^{n} a_{ik} a_{ki'} + \sum_{k=0}^{n} b_{ik} b_{ki'} = 0 \quad -(1)$$

Now, $a_{ki'} = \overline{a_{ik}}$

from (1) we

$$\sum_{k=0}^{n} |a_{ik}|^2 + \sum_{k=0}^{n} |b_{ik}|^2 = 0$$

$$\sum_{k=0}^{n} |a_{ik}|^2 + |b_{ik}|^2 = 0$$

As, all term ≥ 0,

then, $a_{ik} = 0$ & $b_{ik} = 0$

As, i & k are arbitrary.

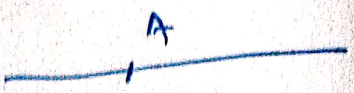
Scanned with OKEN Scanner

Hence, the entire matrix A & B are zero matrix

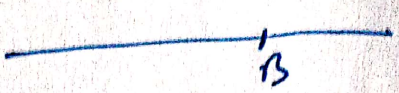
Hence Proved

Scanned with OKEN Scanner

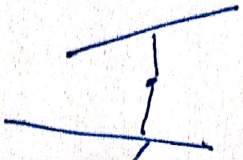
A, B are variable points on two given non intersecting lines and AB is of constant length $2k$. Find the surface generated by AB. (15)



The given lines are



Shew lines



Their general eqⁿ is

let one line be $\vec{a} + \lambda \vec{q}$

& the other be $\vec{c} + \mu \vec{b}$

let the shew lines be

$y = mx, \; z = c$ — (1)

$y = -mx, \; z = -c$ — (2)

(1) $\Rightarrow$

$\frac{x}{l} = \frac{y}{m} = \frac{z-c}{0} = p$

(2) $\Rightarrow$ $\frac{x}{l} = \frac{y}{-m} = \frac{z+c}{0} = q$

let a general point on (1) be

P, mp, c

& on (2)

$q, -mq, -c$

distance is $2k$

$\Rightarrow (p-q)^2 + m^2(p+q)^2 + 4c^2 = 4k^2$

— (3)

Scanned with OKEN Scanner

Candidates must not write on this margin

Eqn of AB is

$$\frac{x-p}{p-q} = \frac{y-mp}{m(p+q)} = \frac{z-c}{2c} = t$$

putting in (3)

we get-

$$(x-p)^2 + (y-mp)^2 + (z-c)^2 = 4k^2t^2$$

Scanned with OKEN Scanner

Candidates must not write on this margin

Show that the lines drawn from the origin parallel to the normals to $ax^2+by^2+cz^2=1$ at its point of intersection with the plane $lx+my+nz=p$ generate the cone $p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$. (15)

let the lines be

$$\frac{x}{ax_1}=\frac{y}{by_1}=\frac{z}{cz_1}=k \quad \text{— (1)}$$

where ax_1, by_1, cz_1 are direction of normal to the conicoid at (x_1, y_1, z_1) let they also lie on the plane.

Now, given

$$ax_1^2+by_1^2+cz_1^2=1 \quad \text{— (2)}$$

&

$$lx_1+my_1+nz_1=p \quad \text{— (3)}$$

Putting $x_1=\frac{x}{ak}$, $y_1=\frac{y}{bk}$, $z_1=\frac{z}{ck}$ in (2) we get-

$$\frac{x^2}{ak^2}+\frac{y^2}{bk^2}+\frac{z^2}{ck^2}=1$$

$$\Rightarrow k^2=\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c} \quad \text{— (4)}$$

Putting in (3) we get

$$\frac{lx}{ak}+\frac{my}{bk}+\frac{nz}{ck}=p$$

$$\Rightarrow k=\frac{1}{p}\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right) \quad \text{— (5)}$$

Putting value of k from (5) in (4) we get-

$$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$$ Hence Proved

Scanned with OKEN Scanner

Q.3 (a) Find the surface generated by a straight line which intersects the lines $x+y=0=z, x-y-z=0=x+y-2a$ and the parabola $y=0=x^2-2az$. (15)

The line lies in

$x+y+\lambda_1 z=0$ — (1)

$x-y-z+\lambda_2(x+y-2a)=0$ — (2)

for some λ_1, λ_2,

Putting $y=0$ & $z=\frac{x^2}{2a}$ we get-

$x+\frac{\lambda_1 x^2}{2a}=0$ — (3)

& $x-\frac{x^2}{2a}+\lambda_2(x-2a)=0$ — (4)

from (3), $x \neq 0$ as $x \neq 0$

$1+\frac{\lambda_1 x}{2a}=0$

$\Rightarrow x=-\frac{1\times 2a}{\lambda_1}=\frac{-2a}{\lambda_1}$

putting in (4)

$-\frac{2a}{\lambda_1}-\frac{4a^2}{\lambda_1^2\times 2a}+\lambda_2\left(-\frac{2a}{\lambda_1}-2a\right)=0$

$\Rightarrow -\frac{2a}{\lambda_1}-\frac{2a}{\lambda_1^2}-2a\lambda_2\left(\frac{1}{\lambda_1}+1\right)=0$

$\Rightarrow \frac{1}{\lambda_1}+\frac{1}{\lambda_1^2}+\lambda_2\left(\frac{1}{\lambda_1}+1\right)=0$

$\Rightarrow \frac{\lambda_1+1+\lambda_2+\lambda_1\lambda_2}{\lambda_1^2}=0$

$\Rightarrow 1+\lambda_1+\lambda_2+\lambda_1\lambda_2=0$ — (5)

Scanned with OKEN Scanner

Candidates must not write on this margin

putting value of λ_1, λ_2 in (5) from (1) & (2) we get

$$1 + -\frac{(x+y)}{2} + \frac{z+y-x}{x+y-2a} + \left(\frac{x+y}{2}\right)\left(\frac{x+y-z}{x+y-2a}\right) = 0$$

$$\Rightarrow \frac{2x+2y-2a-(x+y)(x+y-2a)+z(z+y-x)}{2(x+y-2a)} + \frac{(x+y)(x-y-z)}{2(x+y-2a)} = 0$$

$$\Rightarrow \cancel{2x} + \cancel{2y} - 2a - \cancel{x^2} - xy + 2ax - xy - y^2 + 2ay + z^2 + zy - \cancel{xz} + \cancel{x^2} - xy - xz + yx - y^2 - \cancel{xz} = 0$$

$$\Rightarrow -2a - 3xy + 2ax + 2ay - 2y^2 + z^2 + yz - xz + yx = 0$$

Candidates must not write on this margin

If the axes are rectangular, find the locus of the equal conjugate diameters of the ellipsoid

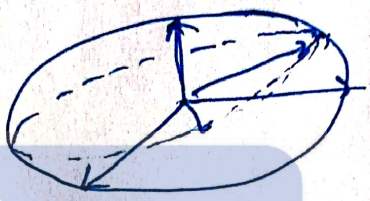
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (20)

let the dc's of the ~~eqn~~ ~~must be~~ conjugate dia be

$(l_1, m_1, n_1), (l_2, m_2, n_2)$ &

(l_3, m_3, n_3)

So,



$\frac{r^2 l_1^2}{a^2}+\frac{r^2 m_1^2}{b^2}+\frac{r^2 n_1^2}{c^2}=1$ — (1)

$\frac{r^2 l_2^2}{a^2}+\frac{r^2 m_2^2}{b^2}+\frac{r^2 n_2^2}{c^2}=1$ — (2)

$\frac{r^2 l_3^2}{a^2}+\frac{r^2 m_3^2}{b^2}+\frac{r^2 n_3^2}{c^2}=1$ — (3)

adding (1), (2) & (3)

$\frac{r^2}{a^2}+\frac{r^2}{b^2}+\frac{r^2}{c^2}=3$ where r = length of conjugate semi diameter.

$\Rightarrow r=\frac{1}{\sqrt{\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)}}$

Hence the ~~conjugate~~ eqn of conjugate diameter

Q.3 (c) Let $T: R^2 \to R^3$ be a linear transformation such that $T(x,y) = (2x+y, x+2y, x+y)$. Let $S: R^3 \to R^2$ be defined by $S(x,y,z) = (-y+2z, y-z)$. Obtain $[S \circ T]_{B_1}$ and verify that $[S \circ T]_{B_1} = [S]_{B_2 B_1} [T]_{B_1 B_2}$, where B_1 and B_2 are standard bases in R^2 and R^3 respectively. (15)

$$T_{B_1 B_2} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{— (1)}$$

$$S_{B_2 B_1} = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{— (2)}$$

Now, $S \circ T = S(\underbrace{2x+y}_{x}, \underbrace{x+2y}_{y}, \underbrace{x+y}_{z})$

$= (-(x+2y)+2(x+y), (x+2y)-(x+y))$

$= (x, y)$ — (3)

Hence $[S \circ T]_{B_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (from (3)) — (4)

Now, $S_{B_2 B_1} \times T_{B_1 B_2}$

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$ from (1) & (2)

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [S \circ T]_{B_1}$ (from (4))

Hence proved

Candidates must not write on this margin

SECTION-B

Find $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$ (10)

$$\lim_{x\to 0}\frac{e^{\ln(1+x)^{\frac{1}{x}}}-e}{x}$$

$$= \lim_{x\to 0}\frac{e^{\frac{\ln(1+x)}{x}}-e}{x}$$

$$= \frac{e^{\lim_{x\to 0}\frac{\ln(1+x)}{x}}-e}{\lim_{x\to 0} x} \quad \text{— (1)}$$

Now, Taylor expansion of $\ln(1+x)$

$$= x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$$

Hence (1) $\Rightarrow$

$$= \lim_{x\to 0}\frac{e^{\frac{x-\frac{x^2}{2}}{x}}-e}{x}$$

$$= \lim_{x\to 0}\frac{e\left(e^{-\frac{x}{2}}-1\right)}{x}$$

$$= \lim_{x\to 0}\frac{e\left(e^{-\frac{x}{2}}-1\right)\times\frac{-1}{2}}{\frac{-x}{2}}$$

$$= -\frac{e}{2} \qquad \left(\text{as } \lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

Scanned with OKEN Scanner

Candidates must not write on this margin

Check for the linear dependence the polynomials $i+x+x^2, -(1+i)-2x+2ix^2, x-x^2$ over

i. $\mathbb{C}$

ii. $\mathbb{R}$ **(10)**

(i) Let

$$\alpha(i+x+x^2)+\beta(-(1+i)-2x+2ix^2)+\gamma(x-x^2)=0$$

$$\Rightarrow x^2(\alpha+2\beta i-\gamma)+x(\alpha-2\beta+\gamma)+(\alpha i-\beta(1+i))=0$$

$\Rightarrow$ $\alpha+2\beta i-\gamma=0$ — (1)

$\alpha-2\beta+\gamma=0$ — (2)

$\alpha i-\beta(1+i)=0$ — (3)

From (3) $\alpha i=\beta(1+i)$

$\alpha=\beta(1-i)$ — (4)

Putting in (2)

$\beta(1-i)-2\beta+\gamma=0$

$\gamma=\beta(2-1+i)=\beta(1+i)$ — (4i)

From (4) ~~Checking consist.~~

Putting in (1)

$\beta(1-i)+2\beta i-\beta(1+i)=0$

$\Rightarrow \beta-\beta i+2\beta i-\beta-\beta i=0$

Hence consistent — (iii)

From (1), (ii) & (iii)

$(\alpha,\beta,\gamma)=\beta(1-i,\ 1,\ 1+i)$ where β can be any number. Hence

in C they are linearly indepen. dependent

(ii) in R

using relationship (1), (2) & (3) from part (i), we get,

$\alpha = \beta(1-i)$ which is not satisfied only for $\alpha = \beta = 0$,

Similarly from other relation $\gamma = 0$ as well,

So, in R, the polynomials are linearly independent.

Candidates must not write on this margin

).5 (c) Integrate the function over the domain $f(x,y) = xy(x^2+y^2)$ over the domain $R:\{-3 \le x^2-y^2 \le 3, 1 \le xy \le 4\}$. (10)

let $u = x^2 - y^2$

$v = xy$

Now $J = \frac{(u,v)}{(x,y)} = \begin{vmatrix} 2x & -2y \\ y & x \end{vmatrix}$

$= 2(x^2+y^2)$

So,

$$\iint_R xy(x^2+y^2)\,dx\,dy$$

$$= \iint_R v \frac{(x^2+y^2)}{J}\,du\,dv$$

$$= \iint_R v \frac{(x^2+y^2)}{2(x^2+y^2)}\,du\,dv$$

$$= \frac{1}{2}\int_{-3}^{3}\int_{1}^{4} v\,dv\,du$$

$$= \frac{1}{2}\int_{-3}^{3}\left[\frac{v^2}{2}\right]_1^4 du$$

$$= \frac{1}{2}\int_{-3}^{3}\frac{15}{2}\,du$$

$$= \frac{1}{2}\times\frac{15}{2}\times 6$$

$$= \frac{45}{2}$$ //.

Scanned with OKEN Scanner

Candidates must not write on this margin

).5 (d) Calculate approximately $(245)^{\frac{1}{5}}$ using LMV theorem. (10)

Consider the function

$f(x) = x^{1/5}$ in the interval $[243, 245]$, it is continuous & differentiable in the interval.

So, By lagrange mean value theorem,

$\exists c \in [243, 245]$ such that

$$f'(c) = \frac{f(245) - f(243)}{245 - 243}$$

$$\Rightarrow f(245) = f(243) + 2f'(c)$$

$$= 3 + 2 \cdot \frac{1}{5} \frac{1}{c^{4/5}} \quad \text{---} (1)$$

~~$= 3.1$~~

For approximation

putting $c = 243$, (1) becomes

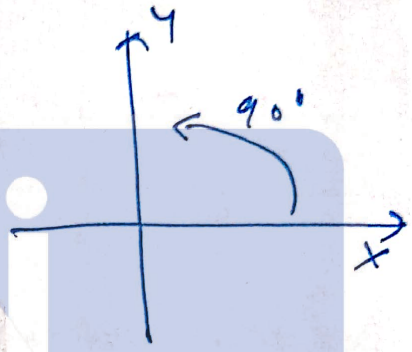
$$f(245) = 3 + \frac{2}{5} \times \frac{1}{81}$$

$$= 3.00494$$

Candidates must not write on this margin

What 3 by 3 matrix represent the transformation that represent the transformation that rotate the xy plane through 90° leaving the z-axis alone? **(10)**

The transformation is given by, the new position of the old basis vector.



Now,

$(1,0,0)$ goes to $\rightarrow (0,1,0)$

$(0,1,0)$ goes to $\rightarrow (-1,0,0)$

$(0,0,1)$ doesn't change.

Hence the transformation matrix is

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} //$$

Scanned with OKEN Scanner

Candidates must not write on this margin

A twice differentiable function is such that $f(a)=f(b)=0$ and $f(c)>0$, for $a<c<b$. Prove that there is at least one value of ξ between a and b for which $f''(\xi)<0$. **(15)**

Ans:

As for all c s.t $a<c<b$,
$f(c)>0$

Let, us pick c_1 such that $f(c_1)>0$

Now, using LMVT between $[a, c_1]$ as well as $[c_1, b]$ we get—

(Note: LMVT conditions are satisfied by given criteria).

$\exists$ c_2 s.t $a<c_2<c_1$ & c_3 s.t $c_1<c_3<b$ which satisfies.

$$f'(c_2)=\frac{f(c_1)-f(a)}{c_1-a}=+ve$$

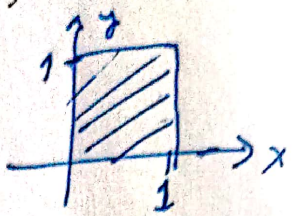
Ily $f'(c_3)=\frac{f(b)-f(c_1)}{b-c_1}=-ve$

Now, using LMVT on the function $f'(x)$ in $[c_2, c_3]$ where the required conditions are satisfied, we have the existence of ξ such that $f''(\xi)=\frac{f'(c_3)-f'(c_2)}{c_3-c_2}$

$=\frac{(-ve)-(+ve)}{+ve}=(-ve)$

Hence Proved

Q.6 (b) Find the maximum and minimum values of $f(x,y)=xy-x^3y^2$ when (x,y) runs over the square $0\le x\le 1, 0\le y\le 1$. (20)



$f(x,y) = xy - x^3y^2$

Here the max & min must be enquired inside as well as on the boundary of the square.

Inside $f_x = y - 3x^2y^2 = 0 \quad y(1-3x^2y) = 0$ — (1)

$f_y = x - 2x^3y = 0 \quad x(1-2x^2y) = 0$ — (2)

So, $(0,0)$ satisfies, if x or y anyone is 0, by virtue of (1) & (2) both are forced to be 0s.

Supposed, both are non zero.

Then, $1-3x^2y = 0$

& $1-2x^2y = 0$

$\Rightarrow x^2y = 0$ which is not possible

Hence, only $(0,0)$ exists which lies on the boundary.

Now, inquiry can be done in 4 parts, for 4 sides

(1) $x = 0$, (3) $x = 1$

(2) $y = 0$ (4) $y = 1$

Candidates must not write on this margin

(1) $x = 0$

$\Rightarrow f(x,y) = 0$

(2) $xy = 0$

$\Rightarrow f(x,y) = 0$

(3) $x = 1$

$f(x,y) = y - y^2$ where $0 \le y \le 1$

For extrem values $\frac{df}{dy} = 0$

$\Rightarrow 1 - 2y = 0$

$\Rightarrow y = \frac{1}{2}$

Here, $f(x,y) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

(4) $y = 1$

$f(x,y) = x - x^3$ where $0 \le x \le 1$

For extrem values $\frac{df}{dx} = 0$

$\Rightarrow 1 - 3x^2 = 0$

$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$

Here $f(x,y) = \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}$

$= \frac{3-1}{3\sqrt{3}} = \frac{2}{3\sqrt{3}}$

Here max value $= \frac{2}{3\sqrt{3}}$ & min value $= 0$

in this rational region

Q.6 (c) Prove that the infinite subset $S = \{1, x, x^2, \ldots\}$, of the vector space P of all real polynomials in x, is linearly independent. (15)

Let's for any m, let -

$$\alpha_0 1 + \alpha_1 x + \alpha_2 x^2 + \ldots \alpha_m x^m = 0$$

putting $x = 0$

we have

$$\alpha_0 + \alpha_1 + \ldots + \alpha_m = 0$$

we have

$$\alpha_0 = 0$$

differentiating both side

$$\alpha_1 + 2\alpha_2 x + \ldots + m\alpha_m x^{m-1} = 0$$

putting $x = 0$

we have

$$\alpha_1 = 0$$

Ily after differentiating m times

we have $\alpha_m = 0$

Here, $\alpha_0 = \ldots = \alpha_m = 0$.

Here, any m of the vector taken in sequential order is independent.

Scanned with OKEN Scanner

Candidates must not write on this margin

If some of the terms are missing in our selection, apart from the highest degree term, the rest of the missing term can be inserted with coefficient of 0. And the same procedure can be carried out.

Hence any finite selection of terms are independt, Hence the set is independent

Proved