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Design of Concrete and Masonry Structures

03
CHAPTER

Working Stress Method (WSM) of Design

3.1 Introduction

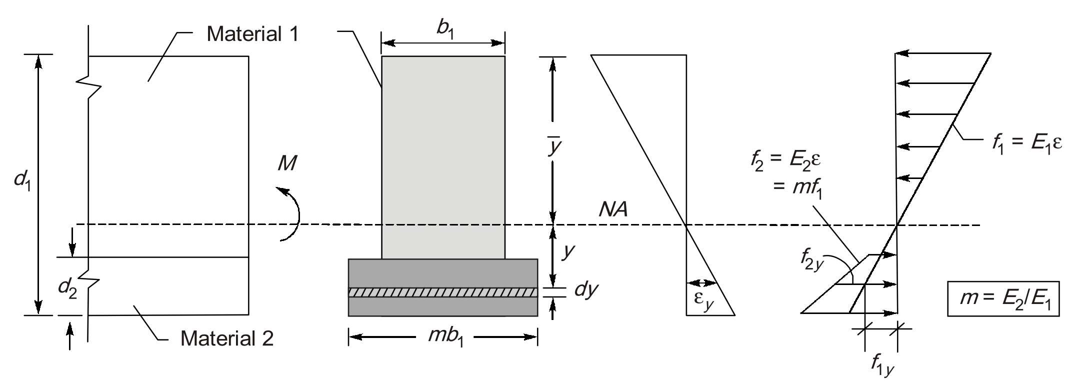
In this chapter, we will discuss the **Working Stress Method** of design which is the most traditional method of design. With the recent advances in the understanding of the behavior of materials (concrete and steel), now we have more rational methods of design. Due to this reason only, most design codes in the world, have dispensed **Working Stress Method.** In the Indian code also i.e. **IS 456 : 2000, Working Stress Method** has been put at annexure and major focus is on the recent **Limit State Method** of design.

WSM finds its application in calculating serviceability requirement like deflection and crack width under service load condition. It is also used in the design of few structures like liquid retaining structures and highway bridges and chimney.

NOTE: Design of water retaining structures and tension structures are not covered by **IS - 456: 2000**.

3.2 Transformed Section

In order to analyse the composite materials by the use of the linear elastic principles of Structural Analysis, it is necessary to transform the composite section into a single homogeneous section. This is made possible by the concept of **modular ratio (*m*)**. In WSM, stress-strain relation is linear and elastic theory is applied. But, elastic theory cannot be applied directly to the non-homogenous section. Transformed section is obtained by replacing the steel area by an equivalent area of same modulus of elasticity as concrete.



(a) beam section (b) strains (c) actual stresses

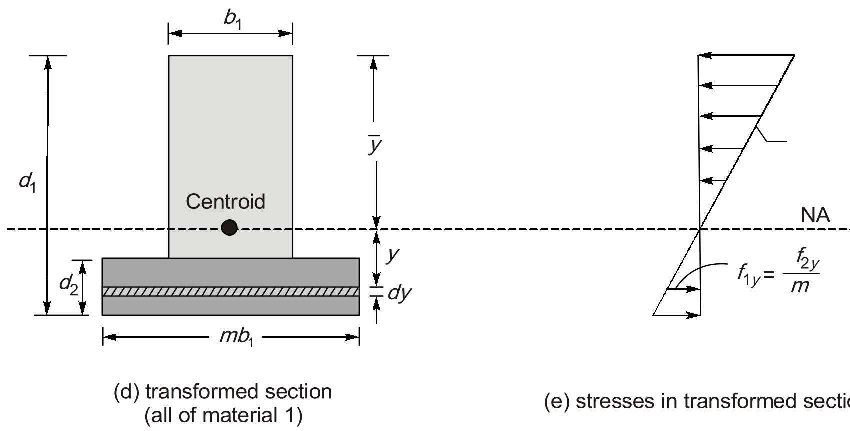


Fig. 3.1 Transformed Section

Let there be an infinitesimal element in material 2 located at a distance 'y' from the neutral axis.

Thus

$$dF_2 = f_2(b_1 \cdot dy)$$

But $$f_2 = \left(\frac{E_2}{E_1}\right) f_1 = mf_1 \text{ as proved earlier}$$

Therefore $$dF_2 = mf_1 (b_1 \cdot dy) = f_1 (mb_1 \cdot dy)$$

Thus material 2 can be transformed to materials 1 by replacing original width b_1 by 'mb_1', where m is the modular ratio $\left(= \frac{E_2}{E_1}\right)$. The term **width** implies dimension parallel to the neutral axis.

NOTE: In the transformed section, the magnitude of resultant forces, their direction and line of action does not change.

In the transformed section, as the material is homogeneous (of one material only), the principles of linear elastic analysis is applicable.

3.3 Modular Ratio

The short term modulus of elasticity of concrete does not take into account the long term effects of creep and shrinkage and thus it is not considered in defining the modular ratio (m). However, partly taking into account the long term effects of creep and shrinkage, **Cl. B-1.3 of IS 456: 2000** defines modular ratio (m) as:

$$\boxed{m = \frac{280}{3\sigma_{cbc}}}$$

or $$m\sigma_{cbc} = \frac{280}{3} = 93.33 = \text{constant}$$

where, σ_{cbc} = Permissible stress in concrete in bending compression

Table 3.1 : Values of σ_{cbc} and m for different grades of concrete

Grade of Concrete	σ_{cbc} (MPa)	Modular Ratio (m)
M15	5	18.67
M20	7	13.33
M25	8.5	10.98
M30	10	9.33
M35	11.5	8.11
M40	13	7.18
M45	14.5	6.44
M50	16	5.83

3.4 Transformed Area of Reinforcement-Compression Steel

The modular ratio for compression steel (eg. Steel in columns, compression steel in doubly reinforced beams) is greater than that for tension steel. This is due to the long term effects of creep and shrinkage of concrete along with non-linearity in material behavior at higher stress levels results in much higher compressive strains in compression steel rather than those indicated by linear elastic theory. Thus **IS 456: 2000** recommends transformed area of compression steel to be equal to 1.5 mA_{sc} and **NOT** mA_{sc}. The corresponding stress in compression steel f_{sc} is given as f_{sc} = 1.5 mf_c. Where, f_c is the corresponding stress in equivalent transformed concrete.

NOTE:
- In tension steel, shrinkage reduces the tensile stress and creep produces additional tensile stresses.
- In compression steel, both shrinkage and creep causes more stress.

3.5 Cracking Moment

The very first crack in the extreme tension fibre of a beam appears when the stress reaches the value of **modulus of rupture of concrete (f_{cr}).** Assuming a linear stress-strain relationship for concrete in compression and tension with the same modulus of elasticity of concrete, the corresponding **cracking moment** is given by:

$$\text{Cracking moment } (M_{cr}) = f_{cr}\frac{I_T}{y_T}$$

where, I_T = Second moment of area or moment of inertia of transformed concrete about neutral axis

y_T = Distance of extreme tension fibre from neutral axis

f_{cr} = $0.7\sqrt{f_{ck}}$ (as per IS code) N/mm^2

f_{ck} is the compressive strength of concrete. The applied moment should have been applied for the 1st time. If a beam section has been applied with a moment greater than M_{cr} and then, it has been unloaded and now it is again being loaded, we will have cracked section even for M < M_{cr}.

When the concrete beam is very lightly loaded such that the applied moment (M) is less than the cracking moment (M_{cr}) then section is **uncracked section,** and both concrete and steel takes part in resisting tension.

3.5.1 The Uncracked Phase in Reinforced Concrete

Initially, there is no externally applied load on the beam. Now gradually, as the beam is being loaded, the corresponding moment at a particular section in the beam increases. When this applied moment at any section of the beam (M) is less than the cracking moment (M_{cr}), then the maximum tensile stress f_T (at the extreme fibre of beam) is less than the flexural tensile strength of concrete f_{cr}. This phase is called as **uncracked phase** and the whole section takes part (i.e. effective) in resisting the applied moment M.

The limiting case of **uncracked phase** occurs when the applied moment (M) becomes equal to the **cracking moment (M_{cr})**.

When the concrete on tension face cracks, it becomes ineffective in resisting tensile stress. Effective area of concrete reduces and tension resisted by concrete prior to cracking is transferred to the steel. With the sudden increase in tension in reinforced steel, there is associated increases in tensile strain in steel bars at the cracked section.

So, this large increase in tensile strain at the steel level, results in an upward shift of the neutral axis.

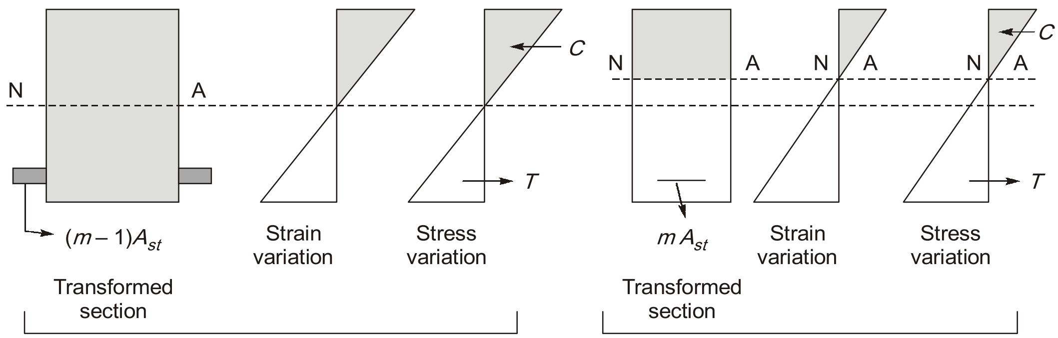
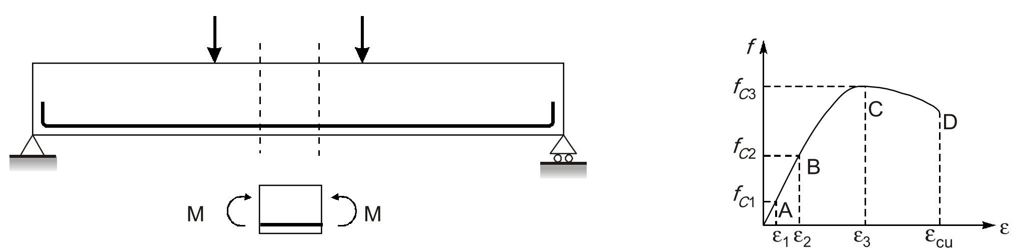


Fig. 3.2

This upward shift of NA and to balance tension and compression, stress and strain in compression have to increase. MOR will increase slightly due to stress transfer from concrete to steel.



(a) Beam with loading (b) Beam with loading

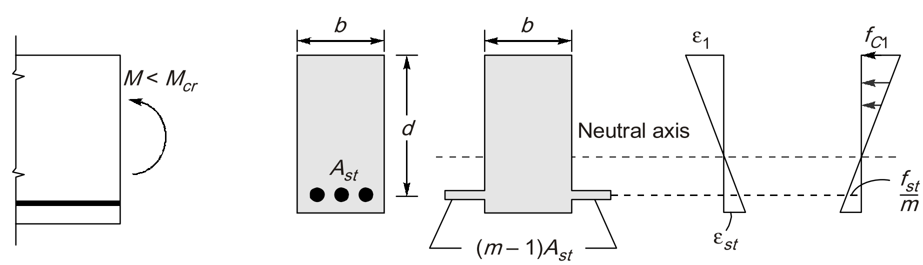


Fig. 3.3 Uncracked phase of concrete

As the externally applied moment M (due to external loading) exceeds the cracking moment M_{cr}, the tensile stress in the concrete exceeds the flexural tensile strength of concrete. At this stage, crack get initiated.

It should be noted that the concrete on the tension side will not become completely useless on account of cracking.

1. They serve the purpose of holding the reinforcement in place.
2. They resist shear and torsion.
3. Provide flexural stiffness so that, deflection is to be controlled.
4. It provides protection to steel against corrosion and fire.

3.6 Permissible Stresses in Concrete and Steel

3.6.1 Permissible Stresses in Concrete and Tension

Cl. B-2.1.1 of IS 456: 2000 specifies permissible stress in direct tension for different grades of concrete. Although full tension is to be taken by reinforcement only, the actual tensile stress in concrete shall not exceed the respective permissible stresses in order to prevent cracks in concrete. Concrete is not assumed to take any tension, the actual tensile stress in concrete is always tried to keep below the permissible stress of concrete in direct tension to avoid cracks in concrete. The factor of safety of concrete in direct tension ranges from 8.5 to 9.5.

Concrete grade	M10	M15	M20	M25	M30	M35	M40	M45	M50
Tensile stress (N/mm^2)	1.2	2.0	2.8	3.2	3.6	4.0	4.4	4.8	5.2

3.6.2 Permissible Stresses in Concrete in Compression

Table 21 of IS 456: 2000 gives the values of permissible stresses in concrete in direct compression, bending compression.

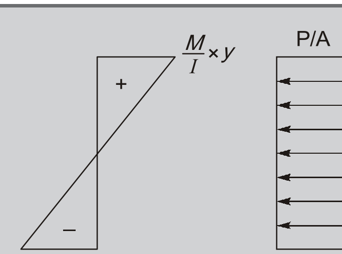
Table 3.2: Permissible Stresses in Concrete (N/mm^2)

Concrete grade	Permissible stress in compression	
	Bending (σ_{cbc})	Direct (σ_{cc})
M10	3.0	2.5
M15	5.0	4.0
M20	7.0	5.0
M25	8.5	6.0
M30	10.0	8.0
M35	11.5	9.0
M40	13.0	10.0
M45	14.5	11.0
M50	16.0	12.0

The factor of safety of concrete in bending compression, direct compression are taken as 3, 4 respectively.

NOTE

- It is observed that for a given grade of concrete $\sigma_{cc} < \sigma_{cbc}$ i.e. greater FOS is adopted for direct stress than for bending stress. Because when a cross-section is subjected to bending stress, the stress induced on it is variable being maximum at extreme fibre and zero at NA.
- When maximum stress exceed the permissible value, extreme fibre will not fail but will transfer the additional force to the inner fibre which have a lower stress, while, the section subjected to direct tensile stress, all points of the section have uniform stress so that there is no scope for force transfer.



3.6.3 Permissible Stress in Steel

Table 22 of IS 456: 2000 specifies permissible stresses in steel reinforcement for various steel grades, bar diameters and types of stress.

In the above table, **FOS for steel = 1.8.** (This is much lower than concrete due to better quality control during the production of steel)

Table 3.3 : Permissible Stresses in Steel (N/mm^2)

Type of stress	Mild steel (Fe 250) (MPa)	HYSD (Fe 415) (MPa)	Fe 500 (MPa)
Tension			
(i) Bar dia upto 20 mm	140	230	275
(ii) Bar dia greater than 20 mm	130	230	275
Compression	130	190	—

For steel reinforcement of smaller dia, the stress will be uniform for direct stress as well as for bending stress. Therefore, in steel bars, the permissible stress in bending and direct stress are the same for dia bars up to 20 mmϕ. For more than 20 mmϕ permissible tensile stress is usually reduced.

The value of σ_{st} is given at the centroid of tensile reinforcement subjected to the condition that when more than one layer of tensile reinforcement is provided, stress at the centroid of outer most layer shall not exceed by more than 10% as that in the given table 3.3.

3.6.4 Increase in Permissible Stresses

Cl. B-2.3 of IS 456: 2000 recommends an increase in the permissible stresses in concrete and steel given in **Table 21 and Table 22** up to a limit of 33.33%. This increase in permissible stresses is made where stresses due to wind loading, seismic forces, temperature loads, shrinkage effects etc. are combined with those due to dead loads, live loads and impact loads. Wind and seismic forces needn't be considered simultaneously.

3.7 Assumptions

The WSM is based on elastic theory of analysis and following assumptions are made as per Cl. B-1.3 of **IS 456: 2000:**

1. Adhesion of concrete to steel is perfect within the elastic limit.
2. At any X-section, plane section before bending, remains plane after bending.
3. Modulus of elasticity of concrete remains the same at all stresses and does not change with the duration of stress.
4. There are no internal stresses in steel when it is embedded in concrete.
5. All tensile stresses are taken up by steel only.

3.8 Singly Reinforced Sections

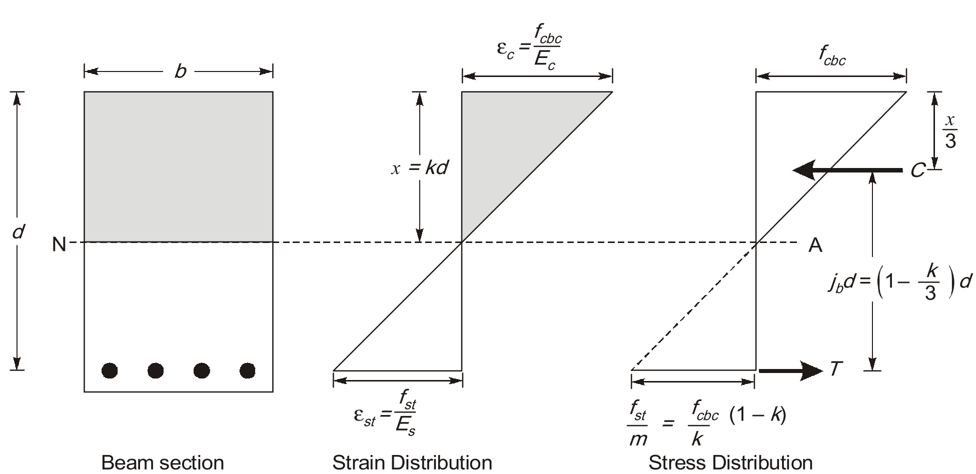


Fig. 3.4 Singly reinforced beam section

The above figure represents strain and stress distribution in a beam section subjected to pure flexure.

where, b = width of beam section, d = Effective depth of beam section

f_{cbc} = Actual stress in concrete in bending compression at the top fibre $\not> \sigma_{cbc}$

f_{st} = Actual stress in steel at the level of centroid of steel reinforcement $\not> \sigma_{st}$

x = kd = depth of neutral axis from the top fibre, k = Neutral axis factor

$jd = \left(1-\frac{k}{3}\right)d$ = Lever arm i.e., distance between the lines of action of compression (C) and tension (T)

Here, stress at the level of centroid of steel reinforcement is $\frac{f_{st}}{m}$ due to transformation of steel into an equivalent area of concrete ($= mA_{st}$)

Neutral Axis

(a) When stress of steel and concrete are given

$$\frac{d-x}{x} = \frac{f_{st}/m}{f_{cbc}} \qquad \text{(from similar } \Delta)$$

$$x = \frac{1}{1+\frac{f_{st}}{m f_{cbc}}} \qquad \text{...(i)}$$

(b) When section dimension and steel area are given

Tension = Compression

$$(mA_{st})\left(\frac{f_{st}}{m}\right) = \frac{1}{2}\times f_{cbc}\times xb$$

$$mA_{st} = \left(\frac{f_{cbc}}{f_{st}/m}\right)\times\frac{1}{2}xb$$

$$mA_{st} = \left(\frac{x}{d-x}\right)\times\frac{xb}{2} \qquad \text{...(ii)}$$

$$\frac{bx^2}{2} = mAst(d-x)$$

i.e. moment of tension area in transformed section about NA = moment of compression area about NA. By solving (ii) equation, x can be calculated.

3.8.1 Singly Reinforced Balanced Section

In balanced section, both f_{cbc} and f_{st} reach their permissible values of σ_{cbc} and σ_{st} respectively at the same time.

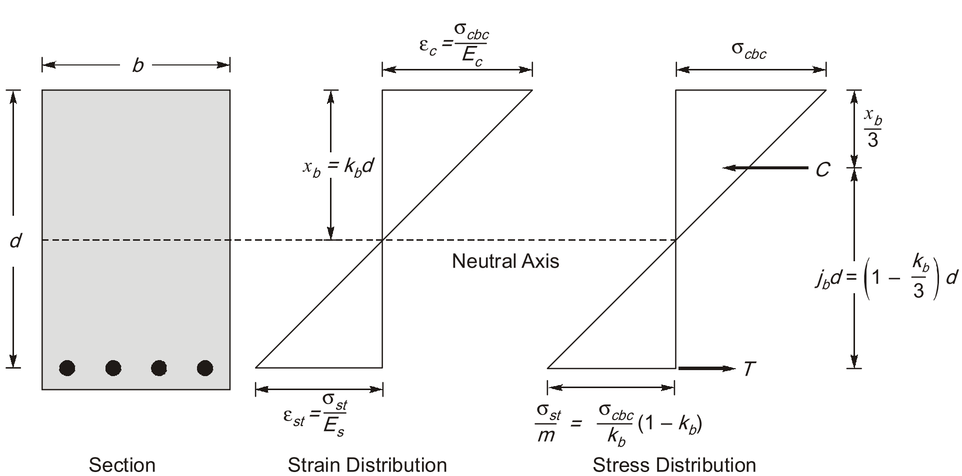


Fig. 3.5

From stress distribution diagram,

$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1-k_b), \quad \text{but} \quad m = \frac{280}{3\sigma_{cbc}}$$

$$\therefore \quad \frac{\sigma_{st}.3\sigma_{cbc}}{280} = \frac{\sigma_{cbc}}{k_b}(1-k_b)$$

$$\Rightarrow \quad k_b = \frac{280}{3\sigma_{st}+280} = \text{Neutral axis factor for balanced section.}$$

Thus the neutral axis for balanced section i.e., $x_b = k_b d$ is known as critical depth of NA.

Also $\quad$ Lever arm $= j_b d = \left(1-\frac{k_b}{3}\right)d$

Total compressive force $= C = \frac{1}{2}\sigma_{cbc} b x_b = \frac{1}{2}\sigma_{cbc} b k_b d$ (It acts at a distance of $x_b/3$ from top)

Total tensile force $= T = \sigma_{st} A_{st}$

Moment of resistance of balanced section

$$\therefore M_b \text{ (with respect to compression)} = C(j_b d) = \frac{1}{2}\sigma_{cbc} b k_b d\left(1-\frac{k_b}{3}\right)d = \frac{1}{2}\sigma_{cbc} k_b j_b \left(d^2 b\right)$$

$$\text{and } M_b \text{ (with respect to tension)} = T(j_b d) = \sigma_{st} A_{st}(j_b d) = \left(\frac{p_{t\,bal}}{100}\right) b\sigma_{st} j_b d^2$$

where, $\quad p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100$ = Percentage of tensile steel for balanced section.

Moment of resistance is also expressed as,

$$M_b = R_b b d^2 \qquad \text{where, } R_b = \frac{1}{2}\sigma_{cbc} k_b j_b = \left(\frac{p_{t\,bal}}{100}\right)\sigma_{st} j_b$$

and lever arm factor $\quad j_b = \left(1-\frac{k_b}{3}\right)$

Also $\quad C = T$

For balanced section, $\quad C = T$ gives,

$$A_{st}\sigma_{st} = \left(\frac{\sigma_{cbc}}{2}\right) b k_b d$$

$$\Rightarrow \quad \boxed{\left(\frac{A_{st}}{bd}\right) = \frac{\sigma_{cbc} k_b}{2\sigma_{st}}}$$

$$\Rightarrow \quad p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100 = \left(\frac{50\sigma_{cbc} k_b}{\sigma_{st}}\right)$$

3.8.2 Under Reinforced Section-Singly Reinforced

Owing to the fact that bar diameters available cannot meet the requirement of steel area required and thus it is not possible to design a balanced reinforced section. Then percentage of steel provided in beams is always kept below the value of '$p_{t\,bal}$' i.e., percentage of tension steel for balanced section.

Salient Features of Under-reinforced Sections

(i) $x_a < x_b$

(ii) Failure of beam is due to failure of reinforcement therefore failure will be ductile.

(iii) Tensile steel reaches the maximum permissible value prior to concrete.

(iv) This type of failure is preferred.

(v) Neutral axis is above the balanced N.A. in under reinforcement section.

(vi) Strain in steel leading to wider and deeper tension cracks and increased beam curvature and deflections.

(vii) The process continues until the maximum strain in concrete in compression side reaches the ultimate compressive strain of concrete resulting in crushing of concrete.

(viii) Section suffers large deflections and cracking prior to failure. Thus, it is a ductile failure.

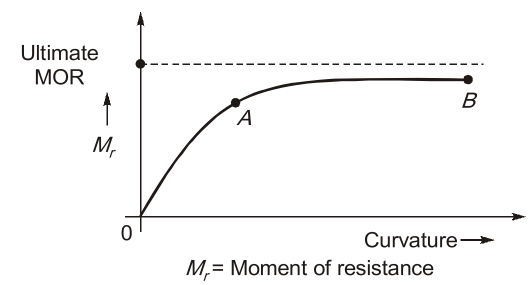


Fig. 3.6

(ix) Ductile failure gives sufficient warning before failure.

(x) MOR beyond MOR balanced (i.e. A) does not increase significantly but curvature increases significantly.

Calculation of MOR

$$x_a < x_b,\ C_a < \sigma_{cbc},\ t_a = \sigma_{st}$$

From compression side

$$\boxed{MR = \frac{1}{2} B x_a C_a \left(d - \frac{x_a}{3}\right)}$$

where

$$C_a = \frac{x_a \sigma_{st}}{(d - x_a)m}$$

For tension side

$$\boxed{MR = \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)}$$

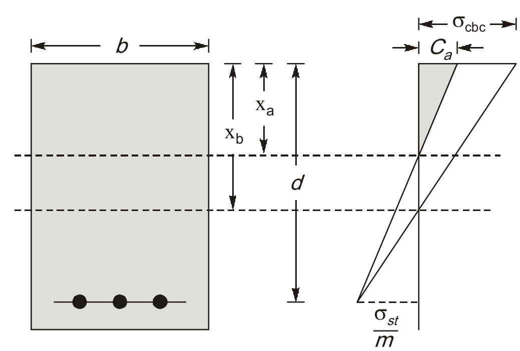
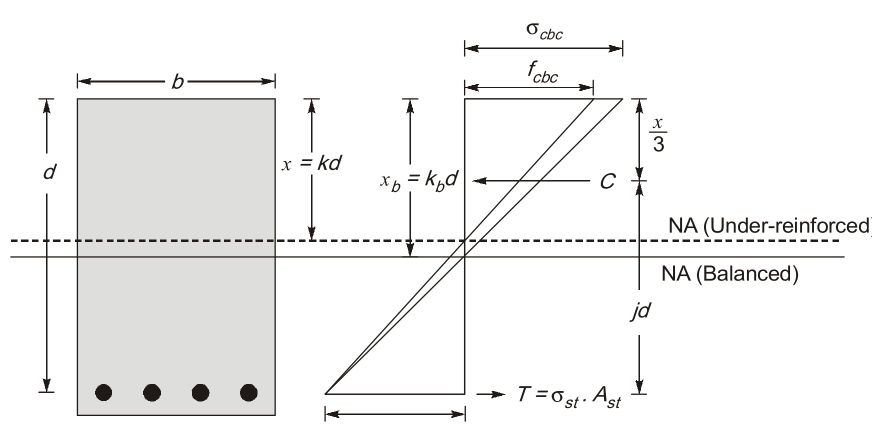


Fig. 3.7



$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1 - k_b)$$

$$= \frac{f_{cbc}}{k}(1 - k)$$

Fig. 3.8

3.8.3 Over-reinforced Sections

Salient feature of over-reinforced section:

(i) $x_a > x_b$

(ii) Failure of beam is due to failure of concrete therefore failure will be Brittle failure.

(iii) Stress in steel is always less than the maximum permissible value.
(iv) This type of sections is not preferred, as there is no significant warning before failure.
(v) Over-reinforced section fails to utilise the full strength of the costlier material.

Calculation of MOR

$$x_a > x_b,\ C_a = \sigma_{cbc},\ t_a < \sigma_{st}$$

From compression side

$$\boxed{MR = \frac{1}{2} b x_a \sigma_{cbc} \left(d - \frac{x_a}{3}\right)}$$

From tension side

$$\boxed{MR = t_a A_{st} \left(d - \frac{x_a}{3}\right)}$$

where

$$t_a = \frac{(d - x_a) m\, \sigma_{cbc}}{x_a}$$

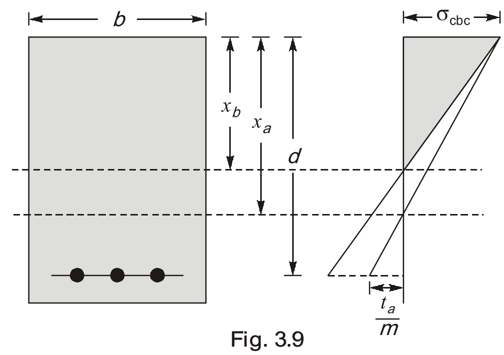


Fig. 3.9

3.9 Doubly Reinforced Beam Section

A doubly reinforced beam is required whenever size of beam is fixed and beam has to resist higher moment (i.e., more than that of MOR of singly reinforced balanced section).

In tensile steel shrinkage reduces the tensile stress and creep produces additional tensile stress but in compression steel both shrinkage and creep and additional stress. So, we have different value of modular ratio.

- Modular ratio for compression steel = 1.5 m
- Critical depth of NA will be same as that of singly reinforced section.

There are two methods to design such beams.

- Increase the concrete mix to increase the capacity of the section.
- Reinforcement are provided in compression zone to give additional strength to the concrete in compression.

Advantage of Compression Steel

- Permits smaller size beams.
- Reduce the long term deflection and increase the ductility of the beam.
- Can be used an anchor bars for positioning the shear r/f.
- Compression reinforcement increases ductilities of beam, they are provided in the seismic zone to withstand stress reversal.

3.9.1 Calculation of NA

$$\frac{bx_a^2}{2} + 1.5mA_{sc}(x_a - d') - A_{sc}(x_a - d') = mA_{st}(d - x_a)$$

...[where d' is effective cover to compression steel]

$$\boxed{\frac{bx_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d') = mA_{st}(d - x_a)}$$

3.9.2 Calculation of MOR

MR = Moment resisted by concrete above NA + Moment resisted by compression reinforcement

MR = $C_1\,(LA)_1 + C_2\,(LA)_2$

$$\boxed{MR = bx_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1)A_{sc}C'(d - d')}$$

NOTE: Hanger bars must not be confused with compression reinforcement bars. Hanger bars are provided to hold stirrups.

3.9.3 Calculation of Stress in Concrete and Steel for Applied Moment

$$\boxed{I = \frac{bx^3}{3} + (1.5\,\text{m} - 1)A_{sc}(x - d')^2 + mA_{st}(d - x)^2}$$

$$\boxed{f_{cbc} = \frac{M}{I} \times x}$$

f_{cbc} = stress in concrete in bending compression

$$\boxed{\frac{f_{sc}}{1.5\,\text{m}} = \frac{M(x - d')}{I}}$$

Find f_{sc} from this, f_{sc} = stress in compression steel

$$\boxed{\frac{f_{st}}{m} = \frac{M(d - x)}{I}}$$

f_{st} = stress in tensile steel

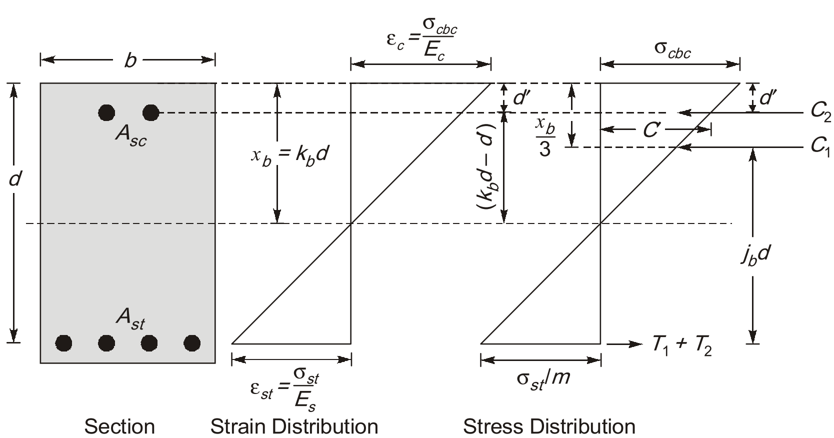


Fig. 3.10

3.10 Limitations of WSM of Design

Use of WSM of design is not only limited to concrete structures but earlier it was used for the design of timber and steel structures also. However, it has the following drawbacks:

1. It is NOT advantageous to use high strength deformed bars as compression reinforcement since the permissible stresses are relatively low and is independent of the grade of steel use as compression reinforcement.
2. In case where large moments are encountered, the area of compression steel (A_{sc}) may even exceed the area of tension steel (A_{st}).
3. It may not be possible to keep the stress within permissible limit. This is because of
   (a) Long term effect of shrinkage and creep.
   (b) Effect of stress concentration and other secondary effect.

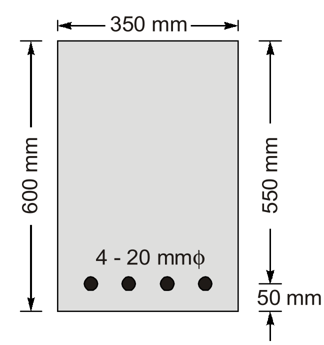
4. In WSM actual margin of safety is not equal to the factor of safety used in WSM because the stress-strain curve is not linear upto collapse. Actual margin of safety here is given in terms of factor like $\frac{\text{Collapse load}}{\text{Working load}}$, the FOS on the other hand is $\frac{\text{Characteristic stress}}{\text{Permissible stress}} = \text{FOS}$.
5. WSM fails to discriminate between various types as loading acting simultaneously, but have different degree of uncertainty. It leads to conservative design specially when two different loads have counteracting effect.

   For Ex.: DL and WL produces counteracting stress but if there are simply added, the design load would be much larger.

EXAMPLE : 3.1

A reinforced concrete beam of size 350 mm × 600 mm (effective cover of 50 mm) is made up of M 20 concrete and reinforced with 4-20 mmϕ of Fe 415 steel. Calculate the cracking moment of the beam and stresses due to an applied moment of 55 kNm. (Take '*m*' for M 20 concrete as 13.33)

Solution:



$$\text{Modular ratio } (m) = 13.33$$

$$\text{Modulus of rupture of M 20 concrete } (f_{cr}) = 0.7\sqrt{f_{ck}}$$

$$= 0.7\sqrt{20} = 3.13 \text{ N/mm}^2$$

$$\text{Section modulus } (Z) = \frac{bD^2}{6}$$

$$= \frac{350 \times 600^2}{6} = 21 \times 10^6 \text{ m}^3$$

$$\therefore \quad \text{Cracking moment } (M_{cr}) \text{ (using gross area)} = f_{cr}.Z = 3.13\,(21 \times 10^6) \text{ Nmm} = 65.73 \text{ kNm}$$

Transformed Section

$$\text{Area of tension steel } (A_{st}) = 4\frac{\pi}{4}(20)^2 = 1256.64 \text{ mm}^2$$

$$\text{Now transformed area } (A_T) = \text{concrete area} + \text{transformed steel area}$$

$$= (A_g - A_{st}) + mA_{st}$$

$$= A_g + (m-1)A_{st}$$

$$= bD + (m-1)A_{st}$$

$$= 350 \times 600 + (13.33 - 1)\,1256.64$$

$$= 225.5 \times 10^3 \text{ mm}^2$$

Depth of NA (y)

Taking moment of transformed area about the top edge

$$A_T y = (bD)\frac{D}{2} + (m-1)A_{st}d$$

$$\Rightarrow \quad y = \frac{\frac{(350)(600)^2}{2} + (13.33-1)1256.64(550)}{225.5 \times 10^3}$$

$$= 317.17 \text{ mm}$$

Thus distance of NA from topmost compression fibre = 317.17 mm = y_c
and distance of NA from tension steel = 550 – 317.17 = 232.83 mm = y_s
distance of NA from bottom most tension fibre = 600 – 317.17 = 282.83 mm = y_t
∴ Second moment of area/MOI of transformed section

$$= I_T = \frac{by_c^3}{3} + \frac{by_t^3}{3} + (m-1)A_{st}y_s^2$$

$$= \frac{350 \times 317.17^3}{3} + \frac{350 \times 282.83^3}{3} + (13.33-1)1256.64(232.83)^2$$

$$= 3722.4 \times 10^6 + 2639.5 \times 10^6 + 840 \times 10^6$$

$$= 7201.9 \times 10^6 \text{ mm}^4$$

∴ Cracking moment (M_{cr}) $= f_{cr}\dfrac{I_T}{y_T} = 3.13 \times \dfrac{7201.9 \times 10^6}{282.83}$ Nmm = 79.7 kNm

Thus cracking moment from gross area (= 65.73 kNm) is under estimated as compared to cracking moment using transformed area (= 79.7 kNm)

Stresses due to applied moment of 55 kNm

Applied moment (M) = 55 kNm < M_{cr}

Thus **uncracked section** analysis can be done.

∴ Maximum compressive stress in concrete

$$= f_c = \frac{M}{I_T}y_c = \frac{55 \times 10^6}{7201.9 \times 10^6} \times 317.17 = 2.42 \text{ N/mm}^2$$

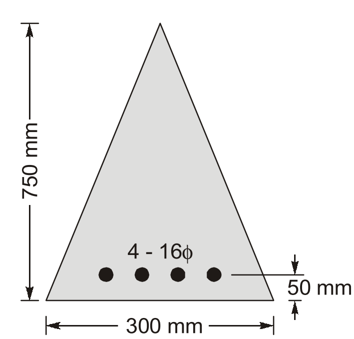
Maximum tensile stress in concrete $= \dfrac{M}{I_T}y_T = f_c\left(\dfrac{y_T}{y_c}\right) = 2.42\dfrac{(282.83)}{317.17}$

$$= 2.16 \text{ N/mm}^2 < f_{cr} (= 3.13 \text{ N/mm}^2)$$

Maximum tensile stress in steel $= f_{st} = mf_c\left(\dfrac{y_s}{y_c}\right) = 13.33(2.42)\dfrac{232.83}{317.17} = 23.68 \text{ N/mm}^2$

EXAMPLE : 3.2

Calculate moment of resistance (MR) of the section as shown in figure below. Concrete grade is M20 and steel is Fe415.



Solution:

σ_{cbc} = 7 N/mm^2, m = 13, σ_{st} = 230 N/mm^2 and d = 700 mm

Limiting depth of neutral axis, $x_c = \dfrac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} \times d = 198.4$ mm

Actual depth of neutral axis, x_a will be calculated from similar triangle as,

$$\frac{b_1}{x_a} = \frac{300}{750}$$

$$b_1 = 0.4\,x_a$$

Equating moment of areas of both the sides about NA

$$\frac{1}{2}b_1 x_a \frac{x_a}{3} = mA_{st}(d - x_a)$$

$$\frac{1}{2}0.4x_a \frac{x_a^2}{3} = 13 \times 804.24(700 - x_a) \qquad \left[\because A_{st} = \frac{\pi}{4}\times(16)^2 \times 4 = 804.24 \text{ mm}^2\right]$$

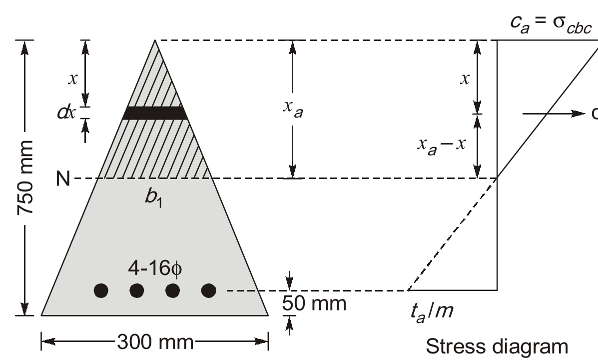
$$x_a = 371.93 \text{ mm} \approx 372 \text{ mm}$$

$$b_1 = 0.4 \times 372 \text{ mm} = 148.8 \text{ mm}$$

$$x_a > x_c$$

It is an over reinforcement section.

$\therefore \qquad t_a < \sigma_{st}$



Let us consider an elementary strip of thickness 'dx' at a distance x from top.

$$\text{Width of strip} = b_x = 0.40x$$

$$\text{Compressive force in strip} = dC = b_x dx C_x$$

Now, from similar triangle in stress diagram

$$\frac{C_x}{(x_a - x)} = \frac{7}{x_a}$$

$$C_x = \frac{(x_a - x)}{x_a}\times 7 = \frac{372 - x}{372}\times 7 = \frac{372 - x}{53.14}$$

Moment of resistance of this elementary strip

$$dM_R = dC \times \text{lever arm} = dC(d - x) = b_x dx\left(\frac{372 - x}{53.14}\right)\times(700 - x)$$

$\therefore$Moment of resistance of the section

$$M_R = \int_0^{x_a} dM_R = \frac{1}{53.14}\int_0^{x_a} 0.4x(700 - x)(372 - x)\,dx = 33.19 \text{ kNm}$$

EXAMPLE : 3.3

Design a simply supported reinforced concrete beam using WSM of an effective span of 7.0 m. The beam is subjected to a live load of 42 kN/m. Width of the beam is 350 mm. Use M30 concrete and Fe500 steel.

Solution:

Given: Effective span (l) = 7.0 m

Width of the beam (b) = 350 mm

Assuming an initial depth of the beam as $\frac{\text{span}}{10} = \frac{7000}{10} = 700$ mm

Self weight of the beam = 0.7 × 0.35 × 25 kN/m = 6.12 kN/m

Live load = 42 kN/m (given)

Total load = 42 + 6.12 kN/m = 48.12 kN/m

Maximum bending moment for simply supported beam (M)

$$= \frac{wl^2}{8} = \frac{48.12 \times 7^2}{8} \text{kNm} = 294.77 \text{ kNm}$$