## ПЕхТ IRS

## QUANTITATIVE APTITUDE

## (Basic Numeracy \& Data Interpretation)

## Comprehensive Study Course <br> CIVIL SERVICES EXAMINATION 2024

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## Quantitative Aptitude

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## QUANTITATIVE APTITUDE

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## UPSC SYLLABUS FOR CSAT

Total Marks : 200

- Comprehension;
- Interpersonal skills including communication skills;
- Logical reasoning and analytical ability;
- Decision making and problem solving;
- General mental ability;
- Basic numeracy (numbers and their relations, orders of magnitude, etc.) (Class X level), Data interpretation (charts, graphs, tables, data sufficiency etc. - Class X level);

Paper-II of the Civil Services (Preliminary) Examination will be a qualifying paper with minimum qualifying marks fixed at $33 \%$. The questions will be of multiple choice, objective type.

## PREFACE

The journey to civil service examinations is one that is filled with dedication, perseverance, and relentless hard work. The Civil Services Aptitude Test (CSAT) is a crucial part of this journey, as it serves as the gateway to the prestigious Indian Civil Services. It is with great pleasure and immense pride that we present to you this book on "Quantative Aptitude" prepared by the NEXT IAS team under the guidance of "Manjul Kumar Tiwari Sir".

The primary aim of this book is to provide aspirants with a thorough understanding of the CSAT examination pattern, the types of questions asked, and the best strategies to solve them. By providing detailed solutions to previous year questions, we hope to instill in you the confidence and ability to tackle any challenge that the CSAT may throw your way.

## 01 Number System

Chapter

## Numerals

A mathematical symbol representing a number in a systematic manner is called a numeral. It is represented by a set of digits

## How to write a Number

To write a number, we put digits from right to left at the places designated as units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs, crores, ten crores and so on.
Let us see how the number 8765432
It is read as:


## Number System

A system in which we study different types of numbers, as well as the relationship and rules that govern them is called a number system
In the Hindu-Arabic system, we use the symbols $0,1,2$, $3,4,5,6,7,8$ and 9 . These symbols are called digits

## Face Value of a Digit

In a numeral, the face value of a digit is the value of the digit itself irrespective of its place in the numeral
Illus. In the numeral 38732 , the face value of 8 is 8 , the face value of 7 is 7 , the face value of 2 is 2 , the face value of 3 is 3 , and so on.

## Place Value (or local Value) of a Digit

In a numeral, the place value of a digit depends on its position in the number.
Illus. Look at the following to get the idea of place value of digits in 213764
Lakhs $\rightarrow$ Place value of $2 \rightarrow 2 \times 100000$
$=200000$

Ten Thousands $\rightarrow$ Place value of $1 \rightarrow$
$1 \times 10000=10000$
Thousands $\rightarrow$ Place value of $3 \rightarrow 3 \times 1000$
$=3000$
Hundreds $\rightarrow$ Place value of $7 \rightarrow 7 \times 100$
$=700$
Tens $\rightarrow$ Place value of $6 \rightarrow 6 \times 10=60$
Units $\rightarrow$ Place value of $4 \rightarrow 4 \times 1=4$
It is clear from the above presentation that to obtain the place value of a digit in a numeral, we multiply the digit with the value of its place in the given numeral

## Classifications of Numbers



## Types of Numbers

## Natural Numbers

- Counting Numbers $1,2,3,4, \ldots .$. upto infinity are called Natural Numbers
- These numbers start with '1'


## Whole Numbers

- All the natural numbers together with '0' are called Whole Numbers


## Integers

- An integer is a number that can be written without a fractional component
- Integers are further classified into :
- Positive integers (1, 3, 4 etc.)
- Zero (0)
- Negative integers (-2, -4 etc.)


## Even Numbers

- All whole numbers divisible by 2 are considered as even numbers. Hence $0,2,4 \ldots$ are even numbers
- The unit place of an even number will be $0,2,4$, 6,8 only
Note: $[-2,-4,-6, \ldots]$ are even integers but they are not even numbers


## Odd Numbers

- All whole numbers not divisible by 2 are odd
- Hence, 1, 3, 5, $7, \ldots$ are odd numbers
- The unit place of an odd number will be 1,3,5, 7 and 9 only
Note: $[-1,-3,-5, \ldots]$ are odd integers. But they are not odd numbers


## Properties of numbers based on Even and Odd

Even + Even $=$ Even
Odd + Even $=$ Odd
Odd + Odd $=$ Even
Odd $\times$ Odd $=$ Odd
Odd $\times$ Even $=$ Even
Even $\times$ Even $=$ Even
$(\text { Even })^{\text {Odd }}=$ Even
$(\mathrm{Odd})^{\mathrm{Even}}=\mathrm{Odd}$
$(\text { Even })^{\mathrm{Even}}=$ Even
$(\text { Odd })^{\text {Odd }}=$ Odd
Ex. There are two, 2-digit numbers ab and cd, if $a b \times c d$ $=273$, what is the sum of $(a b+c d)=$ ?
(a) 21
(b) 35
(c) 47
(d) 66

Sol. (d)
$a b \times c d$ is odd
It means $a b$ and $c d$ both are odd
Hence their sum must be even, only one option is there which is even

## Rational Numbers

- A number which can be expressed in the form of $\frac{p}{q}$ where, p and q are integers and $\mathrm{q} \neq 0$, is called a
rational number. Other examples of rational number are $\frac{-2}{5}, \frac{3}{4}$ etc.
- Rational Numbers have following forms of representations:
(i) Terminating decimal forms

Illus. $0.24=\frac{24}{100}=$ Rational number
(ii) Nonterminating but recurring decimal forms

Illus. 0.121212 ...

$$
\begin{aligned}
& \text { Let } A=0.121212 \ldots \\
& 100 A=12.121212 \ldots \\
& 100 \mathrm{~A}-\mathrm{A}=99 \mathrm{~A}=12 \Rightarrow \mathrm{~A}=\frac{24}{99}=\text { Rational number }
\end{aligned}
$$

Note: Any integer number is rational number since it can be written as the ratio of two integer numbers

## Irrational Numbers

- A real number, which is not rational, is called an irrational number. An irrational number has nonterminating and non-recurring decimal part
- Between any two numbers, there are infinite numbers of irrational numbers
- Examples are : $\sqrt{3}, \sqrt{5}, \sqrt[3]{7}, \sqrt[4]{11}, \pi$ etc.

Note: Any terminating or recurring decimal is a rational number. Any non-terminating and non-recurring decimal is an irrational number

## Real Numbers

- The real numbers include all the measuring numbers
- All the numbers which can be represented on the number line are called real numbers


## Complex Numbers

- All the numbers that can be represented in $a+i b$ form where a and b real numbers and $i=\sqrt{-1}$ are called Complex Numbers
Ex. Which one of the following is not a rational number?
(a) $\frac{4}{7}$
(b) $\frac{-3}{11}$
(c) $\sqrt{3}$
(d) None of these

Sol. (c)
The numbers in option (a) and (b) are rational numbers, as they are the ratio of two integers. The
number $\sqrt{3}$ is non-recurring, so it is not a rational number

Ex. The value of $1 . \overline{34}+4.1 \overline{2}$ is
(a) $\frac{37}{90}$
(b) $\frac{278}{90}$
(c) $3 \frac{278}{99}$
(d) $5 \frac{461}{990}$

Sol. (d)
$1 . \overline{34}$ can be written as
$1 . \overline{34}=1.343434 \ldots .$.
As the bar is placed on two digits after decimal point, so, we will multiply the above equation by $10^{2}$
$1 . \overline{34} \times 10^{2}=134.3434 . \ldots .$.
From equation (i)and (ii)
$\left(10^{2}-1\right) 1 . \overline{34}=133$
$\therefore \quad 1 . \overline{34}=\frac{133}{99}$
In $4.1 \overline{2}$, the bar is place on one digit so it can be written as $4.1 \overline{2}=4.1222 \ldots$.
by multiplying with $10^{2}$ in above equation,
$4.1 \overline{2} \times 10^{2}=412.222 \ldots$
by multiplying with 10 , equation (i) can be
written as $4.1 \overline{2} \times 10=41.222 \ldots$.
From equation (iv) and (v)
$4.1 \overline{2}(100-10)=412-41=412-41$
$\therefore \quad 4 . \overline{1}=\frac{371}{90}$
Hence, the required value is
$1 . \overline{34}+4 . \overline{2}=\frac{133}{99}+\frac{371}{90}$
$=\frac{1330+4081}{990}=\frac{5411}{990}=5 \frac{461}{990}$
Ex. Consider the following statements:
(i) Every whole number is a real number
(ii) Every real number is a rational number
(iii) Every integer is a real number
(iv) Every rational number is a real number

Which of the above statements are correct?
(a) I and II
(b) I, III and IV
(c) I, II and III
(d) II and IV

Sol. (b)
We know that all whole numbers are real numbers but its converse is not true. So, statement (I) is true.
Every real number is not a rational number, some may be irrational numbers. Hence, statement (II) is wrong
Similarly, can say about statement (III) and (IV) that both the statements are true

## Prime Numbers

- Prime numbers are those numbers, which do not have any factor apart from 1 (one) and itself
- Examples of prime numbers are : 2, 3, 5, 7, 11, 13, 17, 19 etc.
- A natural number is called a prime number, if it has exactly two factors, namely 1 and the number itself
- There are 25 prime numbers between 1 to 100
- 2 is the only even number which is prime
- A prime number is always greater than 1
- 1 is not a prime number. Therefore, the lowest odd prime number is 3
- Every prime number greater than 3 can be represented by $6 n \pm 1$, where $n$ is integer. but the converse is not true


## Steps to check whether a given number is prime or not

- Let the number be $n$
- Take the square root of $n$
- If it is a natural number, consider it as it is and if is not a natural number, increase the square root of it to the next natural number
- Now divide the given number by all the prime numbers that lies below the square root obtained
- If the given number is divisible by any of these prime numbers then it is not a prime number else it is a prime number
Illus. Check whether 352 is prime or not?
Sol. Square root of 352 lies between 18 and 19
So, we consider it as 19
Now prime numbers upto 19 are: $2,3,5,7,11,13,17$ and 19
We will divide 352 by all the above prime numbers
$\because 352$ is divisible by 11 so it is not a prime number


## Relative Primes

- Two integers are relative primes or co-primes, if they share no common positive factors (divisors) except 1

Illus. The numbers 4 and 9 do not have any common positive factors (s) hence, they are relative primes. But 4 and 6 are not relative primes

## Composite Numbers

- Numbers greater than 1 which are not prime, are known as composite numbers. They must have atleast one factor apart from 1 and itself
- 1 is neither prime nor composite
- Composite numbers can be both odd and even

Illus. $4,6,8,9,10,12$ etc.

## Co-prime Numbers

- Two natural numbers are said to be co-prime, if their HCF is 1 . Any two consecutive natural numbers as well as any two prime numbers are always co-prime to each other.
- Co-prime numbers may or may not be prime

Illus. $(7,8),(3,5),(4,9)$ etc.

## Perfect Numbers

- If the sum of all the factors of a number, including the factor 1 and excluding the number itself, is equal to the number itself then, that number is called a perfect number.

Illus. $28=1 \times 2 \times 2 \times 7=4 \times 7=2 \times 14$
Hence, the different factors are $(1,2,4,7,14)$ and the sum of these is $1+2+4+7+14=28$.

Other perfect numbers are $6,496,8426$ etc.

## Number Line

Number line is a line on which all the positive and negative numbers can be represented in a sequence. It stretches from negative infinity to positive infinity.


## Operations on Numbers

Let us understand some basic mathematical operations involving numbers

## Addition

- When two or more numbers are combined together, then it is called addition
- It is denoted by ' + ' sign

Illus. $38+43+19=100$

## Subtraction

- When one or more numbers are taken out from another number, then it is called subtraction
- Subtraction is denoted by '-' sign

Illus. $38-14-3=21$

## Multiplication

- When ' $a$ ' is multiplied by ' $b$ ', then ' $a$ ' is added ' $b$ ' times or ' $b$ ' is added ' $a$ ' times. It is denoted by ' $x$ '

Illus. If $\mathrm{a}=4$ and $\mathrm{b}=5$, then $4 \times 5=20$ or $(4+4+4+$ $4+4)=20$
Here, ' $a$ ' is added ' $b$ ' times or in other words 4 is added 5 times
Similarly, $5 \times 4=20$ or $(5+5+5+5)=20$
In this case, ' b ' is added ' a ' times or in other words 5 is added 4 times

## Division

- When $D$ and d are two numbers, then $\frac{D}{d}$ is called the operation of division, where $D$ is the dividend and $d$ is the divisor. A number which tells how many times a divisor (d) exists in dividend $D$ is called the quotient $Q$
- If dividend $D$ is not a multiple of divisor $d$, then $D$ is not exactly divisible by $d$ and in this case a remainder $R$ is obtained

Illus. Let $\mathrm{D}=21$ and $\mathrm{d}=4$
Then, $\frac{D}{d}=\frac{21}{4}=5 \frac{1}{4}$
Here, 5 = Quotient $(Q)$,

$$
4=\text { Divisor }(d)
$$

and $\quad 1=$ Remainder $(R)$
Dividend $=$ Divisor $\times$ Quotient + Remainder

## Fractions

- A fraction is a numerical quantity that is not a whole number
- It represents a part of a whole
- It is expressed in the form as $\frac{\text { numerator }}{\text { denominator }}$

Illus. Find $13 \%$ of 7
Sol. $\quad 13 \%$ of $7=\frac{13}{100} \times 7=\frac{91}{100}$

## Types of Fractions:

Fractions can be broadly classified in 3 types:

1. Proper fractions: Any fraction whose value $<1$
i.e., numerator < denominator
e.g. $\frac{3}{5}, \frac{2}{7}, \frac{19}{23}$ etc.
2. Improper fractions: Any fraction whose value $>1$ ie. numerator $>$ denominator
e.g. $\frac{4}{3}, \frac{17}{11}, \frac{9}{5}$ etc.
3. Mixed fractions: It is another way of writing an improper fraction
It consists of a natural number and a fraction. A mixed fraction is always greater than 1
e.g. $2 \frac{1}{3}, 4 \frac{2}{7}, 3 \frac{8}{11}$ etc.

Note: All the natural numbers can be written in the form of a fraction where denominator is always 1

Ex. Express $\frac{11}{3}$ in mixed fraction.
Sol. $\frac{11}{3}=\frac{9+2}{3}=\frac{9}{3}+\frac{2}{3}=3+\frac{2}{3}=3 \frac{2}{3}$
Ex. Convert $12 \frac{2}{7}$ into improper fraction.
Sol. $12 \frac{2}{7}=12+\frac{2}{7}=\frac{12 \times 7+2}{7}=\frac{86}{7}$

Illus. Solve $\frac{2}{3}+\frac{5}{8}$
Sol. To solve such questions involving fractions, we follow the following steps:

1. Take LCM of denominators.
2. Divide the LCM with each denominator and multiply the quotient with corresponding numerators.
3. Add the resultant numerators.
$\therefore \frac{2}{3}+\frac{5}{8}=\frac{2 \times 8+5 \times 3}{24}=\frac{31}{24}$

Ex. Find the value of $\frac{3}{7}+\frac{4}{9}$
Sol. $\frac{3}{7}+\frac{4}{9}=\frac{9 \times 3+4 \times 7}{7 \times 9}=\frac{27+28}{63}=\frac{55}{63}$
Ex. Solve: $\frac{5 \times \frac{4}{8}+4 \times \frac{3}{7}}{\frac{2}{7} \div \frac{4}{21}}$
Sol. $\frac{5 \times \frac{4}{8}+\frac{4 \times 3}{7}}{\frac{2}{7} \times \frac{21}{4}}=\frac{\frac{20}{8}+\frac{12}{7}}{\frac{3}{2}}=\left(\frac{20}{8}+\frac{12}{7}\right) \times \frac{2}{3}$
$=\left(\frac{140+96}{56}\right) \times \frac{2}{3}=\frac{236}{56} \times \frac{2}{3}=\frac{236}{28 \times 3}=\frac{59}{21}$

## Types of Decimal Fraction

1. Recurring decimal Fraction: The decimal fraction, in which one or more decimal digits are repeated again and again, is called recurring decimal fraction. To represent these fractions, a line is drawn on the digits which are repeated
Illus.
(a) $\frac{1}{3}=0.3333=0 . \overline{3}$,
(b) $\frac{22}{7}=3.142857142857=3 . \overline{142857}$
2. Pure Recurring Decimal Fraction: When all the digits in a decimal fraction are repeated after the decimal point, then the decimal fraction is called as pure recurring decimal fraction
e.g. $0 . \overline{5}, 0 . \overline{489}$ etc.

To convert pure recurring decimal fractions into simple fractions, write down the repeated digits only once in numerator and place as many nines, in the denominator as the number of digits repeating

Illus.
(a) $0 . \overline{6}=\frac{6}{9}=\frac{2}{3}$

Sol. Since, there is only 1 repeated digit.
Therefore, only single 9 is placed in denominator.
(b) $0 . \overline{36}=\frac{36}{99}=\frac{3}{11}$

Sol. Since, there are only 2 repeated digits. Therefore, two 9's are placed in denominator.
3. Mixed Recurring Decimal Fraction: A decimal fraction in which some digits are repeated and some are not repeated after decimal is called as mixed recurring decimal fraction. e.g. $4.1 \overline{2}, 0.1 \overline{23}$ etc.

Illus. $35.5+23.2+43.23=$ ?
Sol. $\quad 35.50$
23.20
$\begin{array}{r}+43.23 \\ \hline 101.93\end{array}$

Ex. $\quad 4.3 \times 0.13=$ ?
Sol. $43 \times 13=559$
Sum of the decimal places $=(1+2)=3$
$\therefore$ Required product $=0.559$

## 'VBODMAS' RULE

To simplify arithmetic expression involving various operations like brackets, multiplication, addition etc., a particular sequence of the operations is followed
The operations have to be carried out in the order, in which they appear in the word stand for following operations
Order of above mentioned operations is same as the order of letters in the 'VBODMAS' from left to right as


First: Vinculum (V) or Bar ${ }^{\prime}$ -
Illus: $-7-11=-18$, but $-\overline{7-11}=-(-4)=4$
Second: Brackets (B)

## Order of removing brackets

1 Small Brackets (Circular brackets) '( )'
2. Middle brackets (Curly brackets) ' $\}$ '
3. Square brackets (Big Brackets) '[ ]'

Third: Operation of 'Of' ( $O$ )
Fourth: Operation of division $-(D)$
Fifth: Operation of multiplication $-(M)$
Sixth: Operation of addition $-(A)$
Seventh: Operation of subtraction $-(S)$
Ex. Solve: $3+8 \times(70 \div 28)+8$ of $\overline{3-1}$
Sol. Following VBODMAS Rule,

$$
\begin{aligned}
& \text { Expression }=3+8 \times(70 \div 28)+8 \text { of } 2 \\
& =3+8 \times(70 \div 28)+16 \\
& =3+8 \times\left(\frac{5}{2}\right)+8 \times 2 \\
& =3+20+16=39
\end{aligned}
$$

Ex. Simplify the following expression:

$$
\frac{5 \times 1.6+4 \div 2-8}{10+6 \times 2-4-\overline{3 \times 8} \div 2}
$$

Sol. Following VBODMAS Rule,

$$
\begin{aligned}
& \text { Expression }=\frac{5 \times 1.6+2-8}{10+6 \times 2-4-24 \div 2} \\
& =\frac{8+2-8}{10+6 \times 2-4-12} \\
& =\frac{10-8}{10+12-4-12}=\frac{2}{22-16}=\frac{2}{6}=\frac{1}{3}
\end{aligned}
$$

## Some Important Algebraic Formulae:

1. $(a+b)^{2}=a^{2}+2 a b+b^{2}$
2. $(a-b)^{2}=a^{2}-2 a b+b^{2}$
3. $(a-b)(a+b)=a^{2}-b^{2}$
4. $(a+b)^{3}=a^{3}+b^{3}+3 a b(a+b)$
5. $(a-b)^{3}=a^{3}-b^{3}-3 a b(a-b)$
6. $a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$
7. $a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$

## Solved Examples

Q. 1 If x is a positive integer and $(x+3)(x+5)$ is odd, then $x(x+2)$ must be a multiple of which one of the following?
(a) 3
(b) 6
(c) 8
(d) 12

Sol. (c)
$(x+3)(x+5)$ is odd only when both $(x+3)$ and $(x+5)$ are odd. This is possible only when $x$ is even. Hence, $x=2 y$, where $y$ is a positive integer. Then,
$x(x+2)=2 y(2 y+2)=2(y) 2(y+1)=4(y)(y+1)=4$ (product of two consecutive positive integers, out of which one must be even) $=4$ (an even numbers), and this equals a number that is at least a multiple of 8 .
Q. 26 is added to a certain number and the sum is multiplied by 4 ; the product is divided by 18 and 3 is subtracted from the quotient. The remainder left is 1 . The number is:
(a) 10
(b) 12
(c) 30
(d) 18

Sol. (b)
Let the number be $x$. Then,
$\Rightarrow \frac{4(x+6)}{18}-3=1$
$\Rightarrow \frac{4(x+6)}{18}=4$
$\Rightarrow x+6=18$
$\Rightarrow \mathrm{x}=12$
Q. 3 A class starts at 10 a.m. and lasts till 1:26 p.m. Three periods are held during this interval. After every period, 10 minutes are given free to the students. The exact duration of each period is:
(a) 48 minutes
(b) 51 minutes
(c) 56 minutes
(d) 62 minutes

Sol. (d)
Time between $10 \mathrm{a} . \mathrm{m}$. and $1: 26 \mathrm{p} . \mathrm{m} .=3 \mathrm{hrs}$. $26 \mathrm{~min} .=3 \times 60+26=206 \mathrm{~min}$

For two periods in between, free time $=2 \times 10$ $=20 \mathrm{~min}$
Remaining time $=(206-20) \mathrm{min}=186 \mathrm{~min}$
Duration of each of the 3 periods $=\frac{186}{3} \mathrm{~min}$
$=62$ minutes
Q. 4 A boy wrote all the numbers from 200 to 300. Then he started counting the number of two's that have been used while writing all these numbers. What is the number that he got?
(a) 110
(b) 115
(c) 120
(d) 130

Sol. (c)
From 200 to 300 there are 101 numbers
There are 100 2's in the hundred place
102 's in tens place, 10 2's in unit place
Thus number of 2 ' $s=100+10+10=120$
Q. 5 If each of $a, b, c$ are divisible by $3(a, b, c \neq 0)$ then abc must be divisible by which one of the following?
(a) 3
(b) 9
(c) 18
(d) None of these

## Sol. (d)

Since each one of the three numbers $a, b$ and $c$ is divisible by 3
Let $a=3 x, b=3 y, c=3 z$, where $x, y, z$ are non-zero integers
$\Rightarrow a b c=27 x y z$
Since $x, y$ and $z$ are integers, $x y z$ is an integer and therefore, $a b c$ is divisible by 27
Q. 6 The difference between the squares of two consecutive even integers is always divisible by:
(a) 3
(b) 5
(c) 4
(d) 8

Sol. (c)
Let the two consecutive even integers be $2 x$ and $2 x+2$

Then, $(2 x+2)^{2}-(2 x)^{2}=(2 x+2+2 x)(2 x+2-2 x)$
$=(4 x+2)(2)$
$=4(2 x+1)$ which is always divisible by 4
Q. 7 The smallest value of $n$, for which $6 n+1$ is not a prime number, is
(a) 1
(b) 2
(c) 3
(d) 4

Sol. (d)
Putting values for $n$,
$n=1,6 n+1=7$
$n=2,6 n+1=13$
$n=3,6 n+1=19$
$n=4,6 n+1=25$ (not a prime)
Q. 8 The product of three consecutive natural numbers, the first of which is an odd number, is always divisible by
(a) 12
(b) 24
(c) 8
(d) 6

Sol. (d)
Let three consecutive numbers be $(2 n+1)$ $(2 n+2)(2 n+3)$. This will always be divisible by 6 .
Q. 9 If $a$ and $b$ are integers and $a-b=$ even , then which of the following is always even?
(a) $a b$
(b) $a^{2}+b^{2}$
(c) $a^{2}+b^{2}+1$
(d) None of these

Sol. (b)
$a-b=$ even
$\Rightarrow$ Both $a, b$ are either odd or even
Let, $a=1, b=3$

$$
\begin{aligned}
a b & =\text { odd } \\
a^{2}+b^{2} & =\text { even } \\
a^{2}+b^{2}+1 & =\text { odd }
\end{aligned}
$$

Let $a=2, b=4$

$$
\begin{aligned}
a b & =\text { even } \\
a^{2}+b^{2} & =\text { even } \\
a^{2}+b^{2}+1 & =\text { odd } \\
a^{2}+b^{2} & =\text { odd }
\end{aligned}
$$

$\Rightarrow a^{2}+b^{2}$ is always even

## Previous Years Solved Questions

Q. 1 A club has 108 members. Two thirds of them are men and the rest are women. All members are married except for 9 women members. How many married women are there in the club?
(a) 20
(b) 24
(c) 27
(d) 30

Sol. (c)
Given,
Total member $=108$
Female member $=108 \times \frac{1}{3}=36$
Given, 9 women are unmarried.
So, number of married women $=36-9=27$
Q. 2 If all numbers from 501 to 700 are written. What is the total number of times does the digit 6 appears?
(a) 138
(b) 139
(c) 140
(d) 141

## Sol. (c)

For numbers between 600 to 700 :
Number of 6 at the units place $=10$
Number of 6 at the tens places $=10$
Number of 6 at hundredth place $=100$
For number between 501 to 599:
Number of 6 at the units place $=10$
Number of 6 at the tens places $=10$
Hence, total number of 6 between (501-700)

$$
=10+10+100+10+10=140
$$

Q. 3 A person is standing on the first step from the bottom of a ladder. If he has to climb 4 more steps to reach the middle step, low many steps does the ladder have.
(a) 8
(b) 9
(c) 10
(d) 11
[UPSC-2016]
Sol. (b)
As per the question:

- he is on the first step
- need four step to get to the middle rung of the ladder.
- Thus the $1+4=5^{\text {th }}$ rung $/$ step is the middle of the ladder.
It means there would be 9 steps in the ladder.
Options (b) is correct.
Q. $4 X$ and $Y$ are natural numbers other than 1 , and $Y$ is greater than X . Which of the following represents the largest number?
(a) $X Y$
(b) $X / Y$
(c) $Y / X$
(d) $(X+Y) / X Y$
[UPSC-2018]
Sol. (a)
$\mathrm{Y}>\mathrm{X}>1$
$X / Y>X / Y$ as $X / Y<1$
$X Y>Y / X$ as $Y / X<Y$ and $X Y>Y$
$X Y>\frac{X+Y}{X Y}$ as $X Y>\frac{X}{Y}+\frac{1}{X}$ as $\frac{1}{Y}+\frac{1}{X}<2$
but $X Y>2$.
Q. 5 If $x-y=8$, then which of the following must be true?

1. Both $x$ and $y$ must be positive for any value of $x$ and $y$.
2. If $x$ is positive, $y$ must be negative for any value of $x$ and $y$.
3. If $x$ is negative, $y$ must be positive for any value of $x$ and $y$.
Select the correct answer using the code given below.
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2 nor 3
[UPSC-2018]
Sol. (d)

$$
\begin{array}{ll}
x=10 & y=2 \\
x=-1 & y=-9 \\
y=-3 & x=5
\end{array}
$$

If $x>0, y$ can be both $>0$ or $<0$
If $x<0, y<0$
Q. 6 The number of times the digit 5 will appear while writing the integers from 1 to 1000 is
(a) 269
(b) 271
(c) 300
(d) 302
[UPSC-2019]
Sol. (c)
Q. 7 Let $\mathrm{p}, \mathrm{q}, \mathrm{r}$ and s be natural numbers such that $\mathrm{p}-2016=\mathrm{q}+2017=\mathrm{r}-2018=\mathrm{s}+2019$ Which one of the following is the largest natural number?
(a) p
(b) q
(c) r
(d) s
[UPSC-2020]
Sol. (c)
$p-2016=q+2017$
$p=q+2017+2016$
Similarly,
$r=q+2017+2018$
$r=s+2018+2019$
Clearly, $r>s, r>q$
Also, $r=p+2$
Therefore, $r$ is largest
Hence (c)
Q. 8 Let A3BC and DE2F be four-digit numbers where each letter represents a different digit greater than 3 . If the sum of the numbers is 15902 , then what is the difference between the values of $A$ and D?
(a) 1
(b) 2
(c) 3
(d) 4
[UPSC-2020]
Sol. (c)
Since A, B, C, D, E, F are all $>3$
$C+F=12$
$B+2=9, B=7$
$\mathrm{E}+3=8$
$\mathrm{E}=5$
A $+\mathrm{D}=15$
Since all variables are distinct and $>3$
A and D can only be 6 and 9
Difference of A and $\mathrm{D}=3$
Hence (c)
Q. 9 How many pairs of natural numbers are there such that the difference of whose squares is 63 ?
(a) 3
(b) 4
(c) 5
(d) 2
[UPSC-2020]
Sol. (a)
Let the required pair of natural numbes is $x$ and $y$
ATQ, $x^{2}-y^{2}=63$
or $(x+y)(x-y)=63$
There are three possible cases in which product of two numbers is 63 .
Case 1: $(x+y)=9$ and $(x-y)=7$
Then $\mathrm{x}=8$ and $\mathrm{y}=1$
Case 2: $(x+y)=21$ and $(x-y)=3$
Then $x=12$ and $y=9$
Case 3: $(x+y)=63$ and $(x-y)=1$
Then $x=32$ and $y=31$
Hence, there are three pairs of natural numbes such that the difference of their squares is 63.
Q. 10 Integers are listed from 700 to 1000. In how many integers is the sum of the digits 10 ?
(a) 6
(b) 7
(c) 8
(d) 9
[UPSC-2021]
Sol. (d)
Numbers whose sum of digits is 10 are:
$703,712,721,730,802,811,820,901$ and 910
Hence, there are 9 such integers in which the sum of the digits is 10 .
Q. 11 Consider the following statements in respect of two natural numbers $p$ and $q$ such that $p$ is a prime number and q is a composite number:

1. $\mathrm{p} \times \mathrm{q}$ can be an odd number
2. $\frac{q}{p}$ can be a prime number
3. $\mathrm{p}+\mathrm{q}$ can be a prime number

Which of the above statements are correct?
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3
[UPSC-2022]

## Sol. (d)

$p$ is a prime number. So, $p$ can be $2,3,5,7,11,13, \ldots$ q is a composite number. So, q can be $4,6,8,9$, 10, ...
Statement 1: $\mathrm{p} \times \mathrm{q}$ can be an odd number, e.g. $(3 \times 9=27)$. Thus, statement 1 is correct.

Statement 2: $\frac{q}{p}$ can be a prime number, e.g. $\left(\frac{4}{2}=2\right)$. Thus, statement 2 is correct.

Statement 3: $\mathrm{p}+\mathrm{q}$ can be a prime number, e.g. $(3+4=7)$

Thus, statement 3 is correct.
Thus, all the statements 1,2 and 3 are correct

## PRACTICE SET: BASIC QUESTIONS

Q. 1 In a division sum, the quotient is 10 times and the divisor is 20 times the remainder. If the divisor is 60 , find the dividend.
(a) 1603
(b) 1803
(c) 2803
(d) 803
Q. 2 If the difference of two numbers is eight times the smaller number, how many times is the smaller number contained in the bigger number?
(a) 8 times
(b) 9 times
(c) 7 times
(d) 10 times
Q. 3 A multiplication sum having been worked out is partially rubbed out; the figures that remain are one of the multiplicand 99 and the last two digits 87 in the product containing 4 digits, what is the other multiplicand.
(a) 13
(b) 23
(c) 3
(d) 33
Q. 4 What is the divisor when the dividend is 1208 , the remainder 8 and the quotient 120 ?
(a) 20
(b) 10
(c) 180
(d) 200
Q. 5540 chocolates were divided among 50 boys and girls so that each boy got 12 chocolates and each girl 9 chocolates. Find the number of girls.
(a) 10
(b) 20
(c) 30
(d) 40
Q. 6 What is the sum of the following series? $\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\frac{1}{3 \times 4} \ldots \frac{1}{50 \times 51}$
(a) $\frac{50}{51}$
(b) $\frac{8}{51}$
(c) $\frac{101}{51}$
(d) None of these
Q. 7 If $(1+\sqrt{7})^{2}=a+2 \sqrt{b}$, find the value of $a \div b$ ?
(a) 4
(b) 2
(c) 8
(d) 6
Q. 8 If the numerator and denominator of a fraction are exchanged then the product of the two fractions become equal to 1 . The total number of such fractions are:
(a) No such fraction exists
(b) 1
(c) 2
(d) More than 2
Q. 9 The expression $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right) \ldots\left(1+\frac{1}{n}\right)$, simplifies to
(a) $\frac{n}{2}$
(b) $n+1$
(c) $\frac{n+1}{2}$
(d) Can't be determined
Q. 10 Max had to do a multiplication sum. Instead of taking 23 as a multiplicand he took 32 and got the result increased by 315 . The wrong product was :
(a) 805
(b) 1120
(c) 855
(d) Data insufficient
Q. 11 Simplify the expression : $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right) \ldots\left(2-\frac{997}{999}\right)$
(a) $\frac{1001}{999}$
(b) $\frac{1001}{3}$
(c) $\frac{1001}{997}$
(d) Can't be determined
Q. 12 When a two digit odd number is divided by a two digit even number, the quotient is 0.5 , then which of the following will definitely be the ratio of odd and even numbers?
(a) $2: 1$
(b) $1: 2$
(c) $4: 7$
(d) $7: 4$
Q. 13 While solving a mathematical sum, Aman first square a given fraction and then subtracted 20 from it rather than first subtracting 20 from the fraction and then squaring it, but still he got the same result. Find the fraction.
(a) $\frac{21}{2}$
(b) $\frac{3}{8}$
(c) $\frac{14}{3}$
(d) None of these

## Answer key

| 1. | (b) | 2. | (b) | 3. | (a) | 4. | (b) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5. | (b) | 6. | (a) | 7. | (a) | 8. | (d) |
| 9. | (c) | 10. | (b) | 11. | (b) | 12. | (b) |

13. (a)
Q. 1 Three times the first of three consecutive even integers is 4 more than twice the second. The third integer is:
(a) 8
(b) 14
(c) 10
(d) 12
Q. 2 The digit in the unit's place of a number is equal to the digit in the ten's place of half of that number and the digit in the ten's place of that number is less than the digit in unit's place of half of the number by 1 . If the sum of the digits of the number is 7 , then what is the number?
(a) 34
(b) 25
(c) 52
(d) Cannot be determined
Q. 3 The product of the digits of a two-digit number is one half that number. If we add 27 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number.
(a) 24
(b) 63
(c) 42
(d) 36
Q. 4 There is a natural number that becomes equal to the square of a natural number when 100 is added to it and to the square of another natural number when 175 is added to it. Find the number:
(a) 1069
(b) 1296
(c) 1125
(d) 1269
Q. 5 The value of the expression $\frac{12^{3}+8^{3}}{12^{2}+8^{2}-96}$ is:
(a) 12
(b) 144
(c) 20
(d) None of these
Q. 6 When 882 is added to another three digit number $6 a 4$ we get a four digit number $15 b 6$. The value of $(a-b)$ is:
(a) 2
(b) 6
(c) 4
(d) 7
Q. 7 A certain number $N$ when divided by 13 or the 17 , leaves a remainder 0 . Find the number.
(a) 1111
(b) 111111
(c) 11111
(d) 1111111
Q. 8 Find the total number of factors of 480:
(a) 12
(b) 16
(c) 20
(d) 24
Q. 9 The number of even factors of 360 is:
(a) 6
(b) 24
(c) 18
(d) Can't be determined
Q. 10 If $23^{2}-7^{2}=32 a$, then the value of $a^{2}$ is:
(a) 225
(b) 625
(c) 100
(d) 25
Q. 11 If $a+\frac{1}{a}=3$, then the value of $a^{3}+\frac{1}{a^{3}}$ is:
(a) 9
(b) 7
(c) 18
(d) 27
Q. 12 If $a+b+c=0$, then the value of $a^{3}+b^{3}+c^{3}$ is:
(a) 0
(b) 1
(c) $a+b+c$
(d) $3 a b c$
Q. 13 When a two digit number is reversed, then the new number becomes $\frac{4}{7}$ th of the original number. The original number is:
(a) 45
(b) 42
(c) 36
(d) 63

## Answer key

1. (d)
2. (c)
3. (d)
4. (d)
5. (c)
6. (a)
7. 

(b)
8. (d)
9. (c)
10. (a)
11.
(c)
12.
(d)
13. (b)

