

# Geotechnical Engineering

## Civil Engineering

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Comprehensive Theory  
*with* Solved Examples

**Civil Services Examination**



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# Properties of Soil and Classification

## 1.1 BASIC DEFINITIONS

**Soil:** The word Soil originated from a Latin word Solum, which means the upper layer of Earth (dug or plowed). This refers to the loose material lying on the Earth's surface which is formed from the disintegration of rocks and contains organic matter and also supports plant life. The word soil has different meanings in different professions which are as under:

- In agriculture, the above definition is used.
- In geology, it means disintegrated rock material overlying the parent rock.
- In civil engineering, it means an unconsolidated material composed of solid particles which may be organic or inorganic in nature produced by disintegration of rocks.

**Difference between rock and soil:** Both rock and soil consists of mineral grains but the bond between mineral grains in soil is much weaker as compared to the bond in rock.

*Terzaghi* defined soil as "natural aggregates of mineral grains which can be separated by a gentle mechanical means like agitation of water etc."

**Soil mechanics:** It is the branch of Civil Engineering which deals with the application of the laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rock.

**Rock mechanics:** It is the science that deals with the application of principles of mechanics to understand the behavior of rock masses. It has been developed because of situations where heavy loads from the structures on the ground have to be transferred to the rock below and also in situations involving underground structures.

**Soil engineering:** It is the branch of engineering which deals with the application of principles of soil mechanics to the engineering problems. It includes site investigations, design and construction of foundations, earth retaining structures and earth structures.

**Foundation engineering:** It is the part of soil engineering that deals exclusively with foundation of structures on the soils.

**Geotechnical engineering:** It has a much wider scope and refers to all the engineering problems involving soil, rock as foundation as well as construction material. It incorporates the application of principles of soil mechanics, rock mechanics, engineering geology, Soil Engineering and Rock Engineering to problems involving soils and rocks.

**Applications of geotechnical engineering:** Geotechnical engineering finds its application in a wide number of areas like:

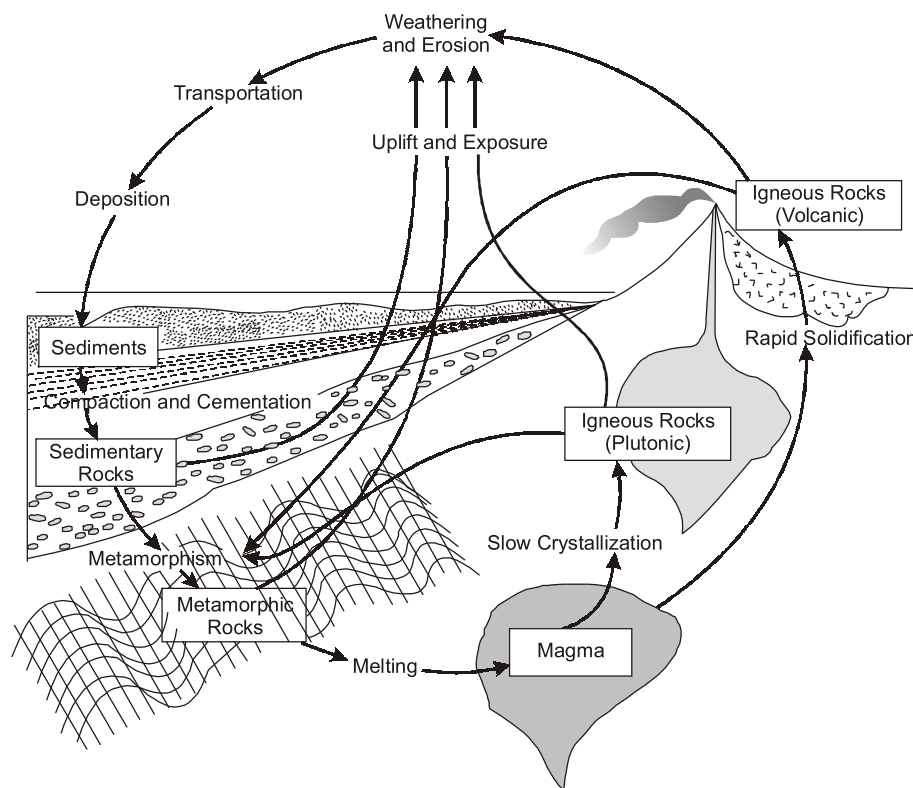
- Shallow foundations
- Underground structures like tunnel
- Embankments and cuts for highway, railway etc.
- Deep foundations
- Earth retaining structures
- Earth and rock fill dams

### 1.1.1 Origin of Soil

Almost all the soils are formed by the disintegration of rocks either through physical, mechanical or chemical weathering. If weathered sediments remain over parent rock, then soil is called 'Residual soil' and weathered sediments transported and deposited at some other place are called 'Transported soil'.

The process of soil formation is called 'Pedogenesis'.

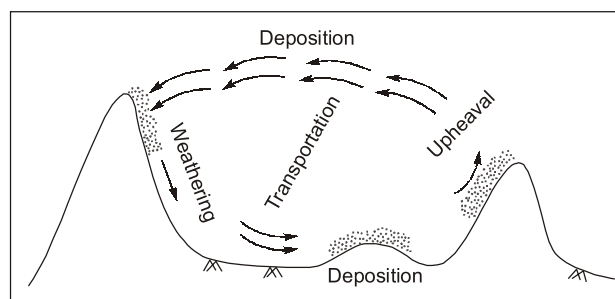
The soil formation is cyclic which is called 'Geological cycle'.



**Fig. The rock cycle**

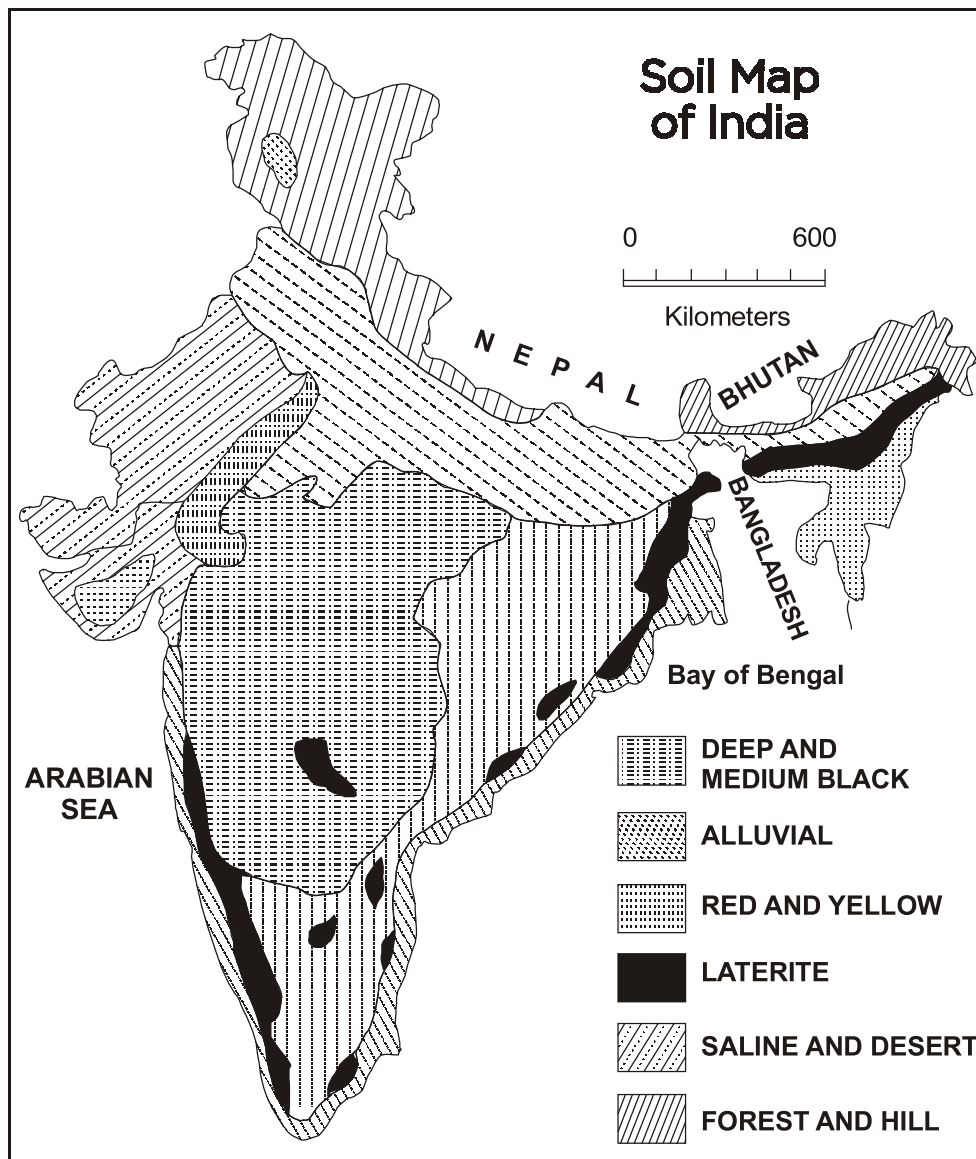
These are the stages in the geological cycle of soil formation in transported soil:

- (1) weathering
- (2) transportation
- (3) deposition of weathered materials
- (4) upheaval



**Fig. Stages of Geological Cycle in case of transported soil**

### 1.1.2 Soil Deposits Commonly Found in India



### 1.1.3 Some of the Most Common Types of Soil

- **Loess:** This is wind blown uniformly graded fine soil. Loess is formed in arid and semi-arid regions. Its colour is yellowish brown and deposits of this soil are found in Rajasthan and North Gujarat.
- **Caliche:** It is cemented soil rich in calcium carbonate consisting of gravel, sand and silt. This is also wind blown in semi-arid climate and later on cemented by the calcium carbonate left out from the evaporation of capillary water.
- **Loam:** It is a mixture of sand, silt and clay in definite proportion which in some cases may consist of organic matter.
- **Cumulose:** Peaty (organic) soil is also called cumulose soil or muck. This is formed due to accumulation of organic content under waterlogged condition. It is generally found in the areas having deficient sewerage facilities or found after overflowing of the rivers.
- **Gumbo:** This is highly sticky, plastic and dark coloured soil.
- **Marl:** This is fine grained calcium carbonated soil of marine origin. This is formed due to decomposition of cell mass and bones of aquatic life.

- **Humus:** This is a mixture of mud and dead plants. The tiny pieces of rock and humus join to make various soils.
- **Peat:** It is highly organic soil containing almost decomposed vegetable matter.
- **Gravel:** It is a type of coarse-grained soil having particle size in the range of 4.75 mm to 80 mm.
- **Sand:** They are cohesionless aggregates of rounded, sub angular or angular sediment in the range of 0.075 mm to 4.75 mm.
- **Silt:** It is a fine-grained soil, with particle size between 0.002 mm and 0.075 mm. The particles are not visible to the naked eyes.
- **Clay:** It is an aggregate of mineral particles of microscopic and submicroscopic range. It may be organic or inorganic.
- **Cobbles:** Cobbles are large size particles (80 mm to 300 mm).
- **Boulders:** Boulders are rock fragment of large size (more than 300 mm).
- **Tuff:** These are small grained slightly cemented volcanic ash that has been transported by wind or water.
- **Bentonite:** It is a clay formed by chemical weathering of volcanic ash which has high content of montmorillonite. Pulverized slurry of bentonite is highly plastic and is often used as a lubricant in drilling.
- **Kaolin (China Clay):** It is a very pure form of white clay, which is extensively used in ceramic industry.
- **Hardpans:** These are the types of soils that offer great resistance to the penetration of drilling tools during soil exploration. These are generally dense, well graded, cohesive aggregates of mineral particles.
- **Varved Clays:** These are sedimentary deposits consisting of alternate thin layer of silt and clay. These clays are the result of deposition in lakes during periods of alternate high and low waters.
- **Till:** It is formed by glaciers and iceberg and may contain mixture of gravel, sand, silt and clay. It is a well grade soil.

## 1.2 PHASE DIAGRAM

- A soil mass is an aggregate of soil particles (with or without water) forming a porous structure. Soil particles in the soil mass are called as soil solids. The pores in the soil mass are called as voids. These voids may be filled with air, water or both.
- The diagrammatic representation of the different phases in a soil mass is called the 'phase diagram', or 'block diagram'.
- Different phases present in soil mass cannot be separated. For better understanding, all three constituents are assumed to occupy separate spaces as shown in figure below.
- Soil mass, in general is a three phase system composed of solid, liquid and gaseous phase.
- A three-phase diagram is applicable for a partially saturated soil ( $0 < S < 1$ )
- When all the voids are filled with water, the sample becomes saturated and thus the gaseous phase is absent; whereas, in oven dry soil sample the liquid phase is absent. Thus, in saturated and oven dry soils, the three phase system reduces to two phase system.

### 1.2.1 Three Phase System

**Three phase system:** A partially saturated soil mass consists of soil solids, water and air. These three physical states are idealized as three separate phases with soil solids in the bottom, water in the middle and air at the top as shown in figure. On one side of the three phase system, weight of solids ( $W_s$ ), weight of water ( $W_w$ ) and weight of air ( $W_a$ ) (which is negligible) are shown and on the other side is shown the volume of solids ( $V_s$ ), volume of water ( $V_w$ ) and the volume of air ( $V_a$ ).

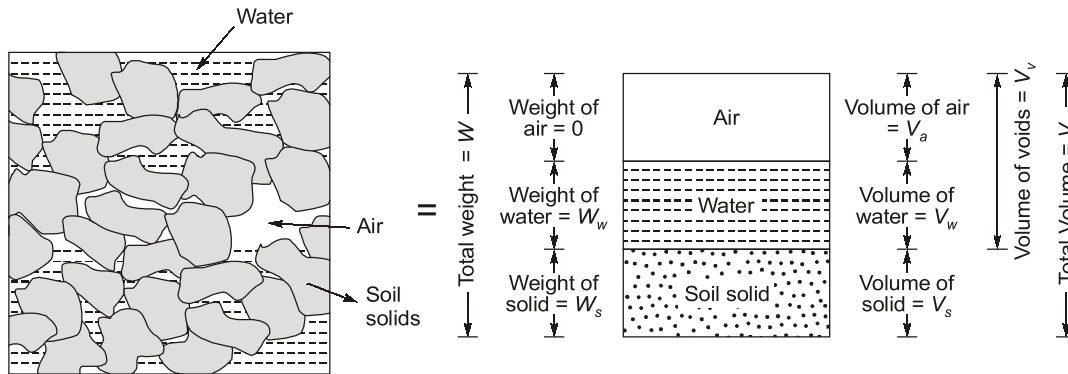


Fig. Three Phase diagram

### 1.2.2 Two Phase System

In two phase system, soil can either be fully dry (with soil solids and air only) or fully saturated (with soil solids and water only).

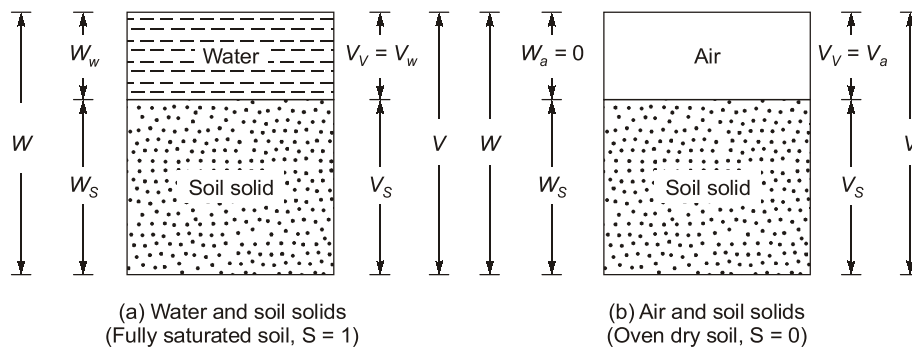


Fig. Two Phase diagram

From the above figures, it can be deduced that:

$$\begin{aligned}
 V &= V_s + V_v \\
 V_v &= V_w + V_a \\
 W &= W_s + W_v \\
 W_v &= W_w \quad (\text{Since } W_a \simeq 0)
 \end{aligned}$$

### 1.2.3 Basic Soil Terminologies

1. **Void ratio:** It is the ratio of volume of voids to the volume of solids and is denoted by  $e$ .

Thus, 
$$e = \frac{V_v}{V_s}$$

Theoretically value of void ratio can vary from zero (when  $V_v = 0$ ) to infinite (when  $V_s \rightarrow 0$ ). Although the individual void sizes in coarse grained soils is more, still for coarse grained soils void ratio varies from 0.5 to 0.9 and for fine grained soils, it varies from 0.7 to 1.5.

2. **Porosity:** It is the ratio of volume of voids to the total volume of soil mass and is denoted by  $n$ .

Thus, 
$$n = \frac{V_v}{V}$$

Since volume of voids ( $V_v$ ) can vary from zero to total soil volume ( $V$  when  $V_s \rightarrow 0$ ) and thus value of porosity ( $n$ ) lies between zero and one. It is usually expressed in percentage.

**REMEMBER**

*Both void ratio and porosity are the measures of denseness or looseness of the soil. As the soil becomes denser, their value decreases because a denser soil implies lesser voids.*

3. **Degree of saturation:** It is the ratio of volume of water to the volume of voids in the soil mass and is denoted by  $S$ .

Thus, 
$$S = \frac{V_w}{V_v}$$

As volume of water occupies the volume of voids and thus volume of water cannot exceed the volume of voids. Therefore the degree of saturation always lies between zero and one (or zero and 100 in percentage).

For dry soil,

$$S = 0$$

For fully saturated soil,

$$S = 1 \text{ or } 100\%$$

4. **Air content:** It is the ratio of volume of air to the volume of voids present in the soil mass and is denoted by  $a_c$ . Thus,

$$a_c = \frac{V_a}{V_v}$$

But,

$$V_a = V_v - V_w$$

Thus,

$$a_c = \frac{(V_v - V_w)}{V_v} = 1 - S$$

5. **Percentage air voids:** It is the ratio of volume of air present in the soil mass to the total volume of soil mass and is denoted by  $n_a$ .

Thus, 
$$n_a = \frac{V_a}{V}$$

But,

$$n = \frac{V_v}{V} \quad \dots(i)$$

And,

$$a_c = \frac{V_a}{V_v} \quad \dots(ii)$$

Multiplying (i) and (ii),

$$n \cdot a_c = n_a$$

6. **Water content:** It is the ratio of weight of water in the soil mass to the weight of soil solids and is denoted by  $w$ .

Thus, 
$$w = \frac{W_w}{W_s}$$

Theoretically water content can vary from zero (when  $W_w = 0$  in case of dry soil) to infinity (when  $W_s \rightarrow 0$  i.e. soil contains too much water as compared to soil solids). Fine grained soils have more water content than coarse grained soils.

### 1.2.4 Densities and Unit Weights

1. **Bulk density:** It is the ratio of total mass of soil to its bulk volume and is denoted by  $\rho$ .

Thus, 
$$\rho = \frac{M}{V}$$

2. **Bulk unit weight:** It is the ratio of total weight of soil mass to its bulk volume and is denoted by  $\gamma$ .

Thus, 
$$\gamma = \frac{W}{V}$$

Now, 
$$W = Mg$$

Thus, 
$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho g$$

3. **Dry density:** It is the ratio of mass of soil solids to the bulk volume of soil and is denoted by  $\rho_d$ .

Thus, 
$$\rho_d = \frac{M_s}{V}$$

As the soil can shrink on drying, so the dry density may not be equal to the bulk density of soil in dried condition.

4. **Dry unit weight:** It is the ratio of weight of soil solids to the bulk volume of soil and is denoted by  $\gamma_d$ .

Thus, 
$$\gamma_d = \frac{W_s}{V}$$

But, 
$$W_s = M_s \cdot g$$

Thus, 
$$\gamma_d = \frac{W_s}{V} = \frac{M_s g}{V} = \rho_d g$$

5. **Saturated density:** It is the ratio of saturated mass of soil  $M_{\text{sat}}$  to the bulk volume of the soil and is denoted by  $\rho_{\text{sat}}$ .

Thus, 
$$\rho_{\text{sat}} = \frac{M_{\text{sat}}}{V}$$

For fully saturated soil mass,  $\rho = \rho_{\text{sat}}$

So, saturated density is the bulk density of the soil when the soil is fully saturated.

6. **Saturated unit weight:** It is the ratio of weight of fully saturated soil mass  $W_{\text{sat}}$  to the bulk volume of the soil and is denoted by  $\gamma_{\text{sat}}$ .

Thus, 
$$\gamma_{\text{sat}} = \frac{W_{\text{sat}}}{V}$$

For fully saturated soil mass,  $\gamma = \gamma_{\text{sat}}$

7. **Specific gravity of soil particles:** It is defined as ratio of weight of a given volume of soil particles to the weight of an equivalent volume of water at a given temperature. Alternatively, it can be defined as ratio of unit weight of soil particles to unit weight of pure water at a given temperature. It is denoted by  $G_s$  or  $G$ .

Thus, 
$$G = \frac{W_s}{V_s \gamma_w} = \frac{\gamma_s}{\gamma_w} \text{ or } \frac{\rho_s}{\rho_w}$$

8. **Specific gravity of soil mass:** It is defined as ratio of bulk unit weight of soil mass to unit weight of pure water at a given temperature. It is denoted by  $G_m$ .

Thus, 
$$G_m = \frac{W}{V \gamma_w} = \frac{\gamma}{\gamma_w} \text{ or } \frac{\rho}{\rho_w}$$

- 9. Submerged density:** It is the ratio of submerged mass of the soil to the bulk volume of the soil and is denoted by  $\rho_{\text{sub}}$  or  $\rho'$ .

When soil mass exists below water level, it is in a submerged condition. A buoyant force acts on the soil solids in this condition which is equal to the magnitude of mass of water displaced by the soil solids.

Thus, Buoyant force,

$$U = \rho_w \cdot V_s$$

So, submerged mass of soil,  $M_{\text{sub}} = M_s - U$

$$= V_s(G\rho_w) - V_s \rho_w$$

$$\left( \because G = \frac{\rho_s}{\rho_w} \right)$$

So,

$$\rho_{\text{sub}} = \frac{V_s \rho_w (G - 1)}{V}$$

Alternatively, considering the equilibrium of entire volume,

$$M = M_{\text{sat}} = M_{\text{st}} V_v \cdot \rho_w$$

$$U = V \rho_w$$

So,

$$\rho' = \frac{M_{\text{sub}}}{V} = \frac{(M_s + V_v \rho_w) - V \rho_w}{V}$$

or

$$\rho' = \frac{M_{\text{sat}}}{V} - \rho_w$$

$\Rightarrow$

$$\rho' = \rho_{\text{sat}} - \rho_w$$

- 10. Submerged unit weight:** It is the ratio of submerged weight of the soil to the bulk volume of the soil and is denoted by  $\gamma_{\text{sub}}$  or  $\gamma'$ .

Thus,

$$\gamma_{\text{sub}} \text{ or } \gamma' = \frac{M_{\text{sub}}}{V} = \frac{(M - M_w)}{V} = \gamma - \gamma_w$$

Also,

$$\gamma_{\text{sub}} = \rho_{\text{sub}} \cdot g$$

- 11. Density of soil solids:** It is the ratio of mass of soil solids to the volume of soil solids and is denoted by  $\rho_s$ .

Thus,

$$\rho_s = \frac{M_s}{V_s}$$

It is also known as mass density of solids.

- 12. Unit weight of soil solids:** It is the ratio of weight of soil solids to the volume of soil solids and is denoted by  $\gamma_s$ .

Thus,

$$\gamma_s = \frac{W_s}{V_s}$$

Also,

$$\gamma_s = \rho_s \cdot g$$

## 1.3 INTER-RELATIONSHIPS BETWEEN THE VARIOUS SOIL TERMINOLOGIES

### 1.3.1 Relation between $W_s$ , $W_w$ and $W$

From block diagram,

$$W = W_s + W_w + W_a$$

$$W = W_s + W_w \quad (\because W_a = 0)$$

$$W = W_s \left( 1 + \frac{W_w}{W_s} \right)$$

$$W = W_s(1 + w) \quad (\because \text{Water content, } w = \frac{W_w}{W_s})$$

$$\therefore W_s = \frac{W}{1 + w}$$

### 1.3.2 Relation between $e$ and $n$

We know, Porosity,  $n = \frac{V_v}{V} = \frac{V_v}{V_s + V_v} = \frac{\left( \frac{V_v}{V_s} \right)}{1 + \left( \frac{V_v}{V_s} \right)}$

$$\Rightarrow n = \frac{e}{1 + e} \quad \left( \because e = \frac{V_v}{V_s} \right)$$

Also,  $\frac{1}{n} = \frac{1 + e}{e} = \frac{1}{e} + 1$

$$\Rightarrow \frac{1}{e} = \frac{1}{n} - 1 = \frac{1 - n}{n}$$

$$\Rightarrow e = \frac{n}{1 - n}$$

### 1.3.3 Relation between $e$ , $S$ , $w$ and $G$

We know,

$$\text{Void ratio, } e = \frac{V_v}{V_s}$$

Also,  $e = \frac{V_v}{V_s} = \frac{V_v}{V_w} \times \frac{V_w}{V_s} = \frac{V_v}{V_w} \times \frac{W_w / \gamma_w}{W_s / \gamma_s} = \frac{V_v}{V_w} \cdot \frac{W_w}{W_s} \cdot \frac{G_s \gamma_w}{\gamma_w} = \frac{1}{S} w G_s$

$$e = \frac{w G_s}{S}$$

or  $Se = wG$

### 1.3.4 Relation between $\gamma_t$ , $G_s$ , $e$ , $w$ and $\gamma_w$

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s \left( 1 + \frac{W_w}{W_s} \right)}{V_s \left( 1 + \frac{V_v}{V_s} \right)}$$

But  $\frac{W_w}{W_s} = w$  and  $\frac{W_s}{V_s} = \gamma_s = G_s \gamma_w$

$$\therefore \gamma_t = \frac{G_s \gamma_w (1 + w)}{1 + e}$$

But  $w = \frac{Se}{G_s}$

$$\therefore \gamma_t = \left( \frac{G_s + Se}{1 + e} \right) \gamma_w$$

**Special Case (a):** If soil is saturated, then

$$\gamma_t = \gamma_{\text{sat}} \quad \text{and} \quad S = 1$$

Hence

$$\gamma_{\text{sat}} = \left( \frac{G_s + 1 \times e}{1 + e} \right) \gamma_w$$

or

$$\gamma_{\text{sat}} = \left( \frac{G_s + e}{1 + e} \right) \gamma_w$$

**Special Case (b):** If soil is dry, then

$$\gamma_t = \gamma_d \quad \text{and} \quad s = 0$$

Hence

$$\gamma_d = \left( \frac{G_s + 0 \times e}{1 + e} \right) \gamma_w$$

or

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

**Special Case (c):** If soil is submerged, then

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = \left( \frac{G_s + e}{1 + e} \right) \gamma_w - \gamma_w$$

$$\gamma' = \left( \frac{G_s - 1}{1 + e} \right) \gamma_w$$

### 1.3.5 Relation between $\gamma_t$ , $\gamma_d$ , $w$

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V}$$

$$\gamma_t = \frac{W_s(1 + W_w / W_s)}{V}$$

or

$$\gamma_d = \frac{\gamma_t}{1 + w} \quad \left( \because \gamma_d = \frac{W_s}{V} \right)$$

### 1.3.6 Relation between $\gamma_d$ , $G_s$ , $w$ and $n_a$

$$V = V_s + V_w + V_a$$

$$1 = \frac{V_s}{V} + \frac{V_w}{V} + \frac{V_a}{V} = \frac{V_s}{V} + \frac{V_w}{V} + n_a$$

or

$$1 - n_a = \frac{V_s}{V} + \frac{V_w}{V} = \frac{W_s / G_s \gamma_w}{V} + \frac{w W_s / \gamma_w}{V} \quad \left( \because V_w = \frac{w W_s}{\gamma_w} \right)$$

$$= \frac{\gamma_d}{G_s \gamma_w} + \frac{w W_s / \gamma_w}{V}$$

$$= \frac{\gamma_d}{G_s \gamma_w} + \frac{w \gamma_d}{\gamma_w} = \frac{\gamma_d}{\gamma_w} \left( W + \frac{1}{G_s} \right)$$

or

$$\gamma_d = \frac{(1 - n_a) G_s \gamma_w}{1 + w G}$$

**Special Case (a):** When  $n_a = 0$ , then soil become fully saturated at a given water content

Hence

$$\gamma_d = \frac{G_s \gamma_w}{1 + w G}$$



**EXAMPLE - 1.1**

The mass of moist soil is 25 kg and its volume is 0.02 m<sup>3</sup>. It is dried in an oven and its mass reduces to 18 kg. Determine the water content, density of moist soil, dry density, void ratio, porosity and degree of saturation. Take  $G = 2.7$ .

**Solution:**

Given:  $M = 25 \text{ kg}$ ,  $V = 0.02 \text{ m}^3$ ,  $M_s = 18 \text{ kg}$

Mass of water,  $M_w = 25 - 18 = 7 \text{ kg}$

- Water content,  $W = \frac{M_w}{M_s} = \frac{7}{18} = 0.3889 = 38.89\%$

- Density of moist soil,  $\rho = \frac{M}{V} = \frac{25}{0.02} = 1250 \text{ kg/m}^3$

- Dry density,  $\rho_d = \frac{\rho}{1+w} = \frac{1250}{1+0.3889} = 900 \text{ kg/m}^3$

- Also,  $\rho_d = \frac{G\rho_w}{1+e}$

$$\Rightarrow e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.7 \times 1000}{900} - 1$$

$$\Rightarrow e = 2$$

- Porosity,  $n = \frac{e}{1+e} = \frac{2}{1+2} = 0.6667 = 66.67\%$

- Degree of saturation,  $S = \frac{wG}{e} = \frac{0.3889 \times 2.7}{2} = 0.525 = 52.5\%$



**EXAMPLE - 1.2**

A soil sample has a porosity of 45% and a water content of 15%. The specific gravity of soil solids is 2.70. Determine the water to be added to 150 m<sup>3</sup> of this soil to achieve full saturation.

**Solution:**

Let the volume of solids,  $V_s = 1 \text{ m}^3$

Mass of solids,  $M_s = G\rho_w = 2.7 \times 1000 = 2700 \text{ kg}$

Mass of water,  $M_w = wM_s = 0.15 \times 2700 = 405 \text{ kg}$

$$\text{Volume of water} = \frac{M_w}{\rho_w} = \frac{405}{1000} = 0.405 \text{ m}^3$$

$$\text{Now, void ratio, } e = \frac{n}{1-n} = \frac{0.45}{1-0.45} = 0.8182$$

Volume of voids,  $V_v = eV_s = 0.8182 \times 1 = 0.8182 \text{ m}^3$

So, volume of air =  $V_v - V_w = 0.8182 - 0.405 = 0.4032 \text{ m}^3$

Hence, volume of additional water to be added for full saturation = 0.4032 m<sup>3</sup>

Now, Total volume of soil,  $V = V_v + V_s = 0.8182 + 1 = 1.8182 \text{ m}^3$

$\therefore$  Volume of water required for  $1.8182 \text{ m}^3$  of soil =  $0.4032 \text{ m}^3$

So volume of water required for  $150 \text{ m}^3$  of soil =  $\frac{0.4032}{1.8182} \times 150 = 21.176 \text{ m}^3$

Mass of water required =  $21.176 \times \rho_w = 21.176 \times 1000 = 21176 \text{ kg}$



### EXAMPLE - 1.3

A saturated soil sample has a water content of 20% and a bulk unit weight of  $18 \text{ kN/m}^3$ . Determine dry density, void ratio and specific gravity of solid particles. What will be the bulk unit weight of the same soil at the same void ratio but at a degree of saturation of 70%? Take  $\gamma_w = 10 \text{ kN/m}^3$ .

**Solution:**

$$\gamma_{\text{sat}} = \frac{G\gamma_w}{1+wG}(1+w)$$

$$\Rightarrow 18 = \frac{G \times 10}{1+0.2G}(1+0.2)$$

$$\Rightarrow G = 2.14$$

Now, for saturated soil,  $S = 1$

$$\text{So, } e = wG = 0.2 \times 2.14 = 0.428$$

$$\text{Dry density, } \gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.14 \times 10}{1+0.428} = 14.986 \text{ kN/m}^3$$

For second case,  $e = 0.428$  and  $S = 0.7$

$$\gamma = \frac{(G + Se)\gamma_w}{1+e} = \frac{(2.14 + 0.7 \times 0.428) \times 10}{1+0.428} = 17.08 \text{ kN/m}^3$$



### EXAMPLE - 1.4

A sample of soil coated with paraffin wax was weighed to be 700 gm. The sample was then immersed in water and the volume of water displaced was 370 ml. The mass of sample without wax was 695 gm and the water content of a representative sample was 20%. Determine the bulk density, dry density, void ratio and the degree of saturation. The specific gravity of solids was 2.65 and of wax was 0.89.

**Solution:**

$$\text{Mass of wax} = 700 - 695 = 5 \text{ gm}$$

$$\text{Volume of wax} = \frac{\text{Mass of wax}}{\rho_{\text{wax}}} = \frac{5}{0.89 \times 1} = 5.618 \text{ ml}$$

$$\begin{aligned} \text{Volume of soil} &= \text{Volume of water displaced} - \text{volume of wax} \\ &= 370 - 5.618 = 364.382 \text{ ml} \end{aligned}$$

$$\text{Bulk density, } \rho = \frac{M}{V} = \frac{695}{364.382} = 1.907 \text{ gm/ml}$$